

## A GOOD PROBLEM SOLVER IS...

- CURIOUS**..... Wonder about things, ask questions, explore to find out more!
- FLEXIBLE**..... Look for different ways to solve a problem.
- PERSISTENT**..... Don't give up easily...take time to think and keep on trying.
- A RISK TAKER**... Try new or challenging things...don't be afraid of making mistakes--they are learning opportunities!
- REFLECTIVE**..... Take time to think about what you're doing, why you're doing it, whether it makes sense, and how you can do it better.

## GUIDELINES FOR PROBLEM SOLVING

1. Help each other do their best thinking.
2. Give help or a hint *only* if the learner wants it. (Would you like help? a hint?)
3. Help the learner discover solutions and mistakes. (How did you get that? Show me how you figured it out.)
4. If you disagree with someone's solution or method, ask them how they got it. If you still disagree, tell them why. Politely explain your own thinking.

**On Saturday, I walked in the park and saw 39 tulips blooming. On Sunday I saw 54 more tulips blooming. How many tulips did I see in bloom?**

**After the rainstorm, I saw 100 earthworms on the sidewalk. At recess, I saw only 67 worms. How many worms disappeared?**

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**I was on the 28<sup>th</sup> floor of a very tall building in New York City. I went on the elevator and got out at the 72<sup>nd</sup> floor. How many floors did I travel on the elevator?**

## How Do We Define Fluency?

Fluency, as we use it here, includes three ideas: efficiency, accuracy, and flexibility.

- *Efficiency* implies that the student does not get bogged down to too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily keeping track of subproblems and making use of intermediate results to solve the problem.
- *Accuracy* depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.
- *Flexibility* requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit subtraction. Students need to be flexible in order to choose an appropriate strategy for the problem and hand, and also to use one method to solve a problem and another method to double-check the results.

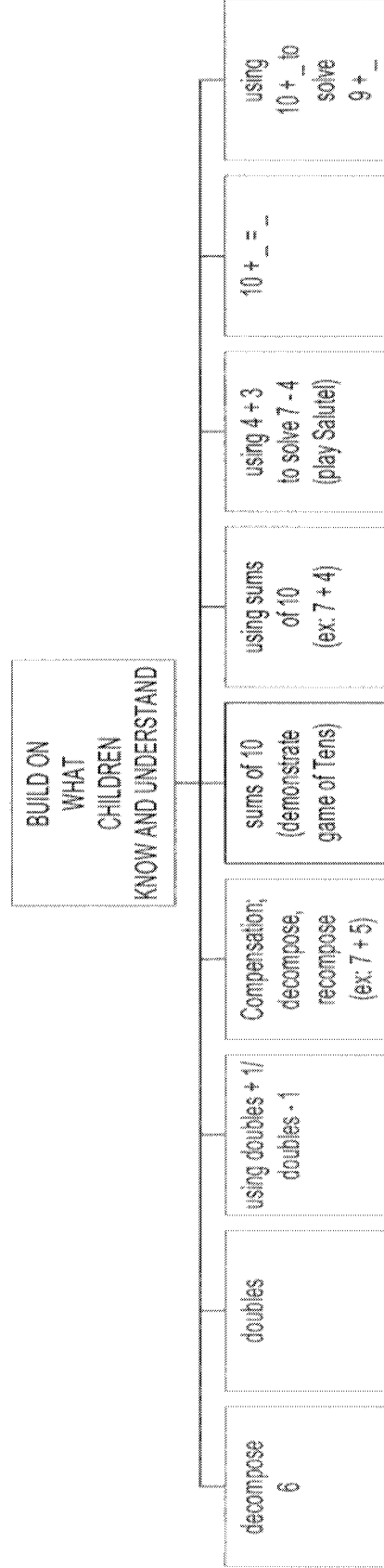
Fluency demands more of students than does memorization of a single procedure. Fluency rests on a well-built mathematical foundation with three parts: (1) an understanding of the meaning of the operations and their relationships to each other – for example, the way addition is related to subtraction; (2) the knowledge of a large repertoire of number relationships, including the addition and multiplication “facts” as well as other relationships, such as how  $4 \times 5$  is related to  $4 \times 50$ ; and (3) a thorough understanding of the base ten number system, how numbers are structured in this system, and how the place value system of numbers behaves in difference operations – for example, that  $24 + 10 = 34$  or  $24 \times 10 = 240$ .

Many alternative (“nonstandard”) algorithms, which are both efficient and accurate, enable students to keep in mind the connections between the numbers and the problem situation. When students work out their own procedures, based on their developing number sense, typically their approaches are connected closely to the meaning of the operations and are based on number relationships they already know.

This excerpt was taken from the book *Relearning To Teach* by Susan Jo Russell. Published by Dale Seymour Productions in Upper Saddle River, NJ.

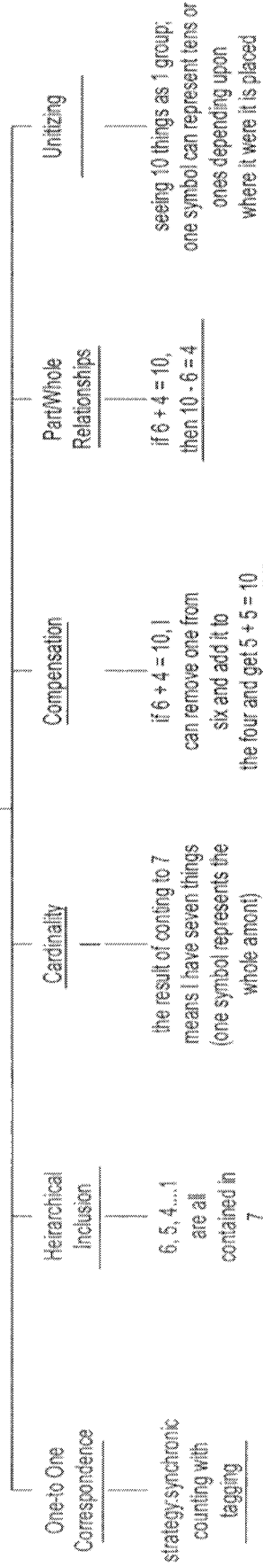
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### DEVELOPING COMPUTATIONAL FLUENCY IN ADDITION AND SUBTRACTION



HANDOUT  
#3

Big Ideas  
Underlying  
Quantity,  
Addition, and Subtraction



Adapted from Cathy Twomey Fosnot and Maarten Dolk, "Young Mathematicians at Work: Construction Number Sense, Addition and Subtraction", NH: (Perismouth, NH: Heineman, 2001).