


Patterns to Symbols: Algebra

Video Note Taking Guide

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Patterns


- ⌘ **All [life] is pattern ... but we can't always see the pattern when we're part of it.**
 - ⌘ *Belva Plain*
- ⌘ **Art is the imposing of a pattern on experience, and our aesthetic enjoyment is recognition of the pattern.**
 - ⌘ *Alfred North Whitehead*
- ⌘ **When will we be done with patterns and go back to math?**
 - ⌘ *Unknown 5th grader*

Why focus on patterns?


- ⌘ **Disappointing results on national comparisons point to our students' difficulty with algebra**
- ⌘ **"The transition from arithmetic to algebra is often not an easy one"** (*Adding It Up*)
- ⌘ **"It is proposed that the artificial separation of arithmetic and algebra deprives students of powerful schemes for thinking about mathematics in the early grades and makes it more difficult for them to learn algebra in the later grades."** (Carpenter & Levi, 2000)

Why focus on patterns? 


- ⚡ **To address this, we need to focus on algebra in the elementary years (This does not mean Algebra 1 in 3rd grade.)**
- ⚡ **Algebraic thinking at the elementary level**
 - ⚡ Making generalizations
 - ⚡ Using symbols
- ⚡ **There are numerous types of generalizations and numerous uses of symbols**

Why focus on patterns? 

- ⚡ **One of the easiest ways to begin to think about teaching generalization is through the use of patterns**
- ⚡ **In NCTM's *Principles and Standards for School Mathematics*, "Understand patterns, relations, and functions" is a standard at all grade levels**

A pattern is a pattern is a pattern 

- ⚡ **Kindergarteners are asked to make patterns and to continue a pattern by coloring**



- ⚡ **So by 3rd grade we better be asking a little more... Making the steps toward generalizing**

NCTM Standards show the progression



⌘ Grades Pre-K–2

- Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another
- Analyze how both repeating and growing patterns are generated

⌘ Grades 3–5

- Describe, extend, and make generalizations about geometric and numeric patterns
- Represent and analyze patterns and functions, using words, tables, and graphs

⌘ Grades 6–8

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

Let's look at some patterns



⌘ 2, 4, 6, 8, ...

⌘ 5, 10, 15, 20, ...

⌘ 5, 8, 11, 14, ...


⌘ 1, 2, 4, 7, 11, ...

2, 4, 6, 8, ...



⌘ **What are the next three numbers in the pattern? How do you know?**

2, 4, 6, 8, ...




⚡ **What are the next three numbers in the pattern? How do you know?**

- William Grade 3

it is counting by 2's The next 3 numbers are 10, 12, 14.

2, 4, 6, 8, ...



⚡ **What are the next three numbers in the pattern? How do you know?**


- William Grade 3

it is counting by 2's The next 3 numbers are 10, 12, 14.

- Claire Grade 6

10, 12, 14, the pattern is plus two each time.

2, 4, 6, 8, ...



⚡ **Would 21 ever be in the pattern? How do you know?**

2, 4, 6, 8, ...



Would 21 ever be in the pattern? How do you know?

- William Grade 3

no [If you look carefully
you can see a pattern
1 is not in the pattern]

2, 4, 6, 8, ...



Would 21 ever be in the pattern? How do you know?

- William Grade 3

no [If you look carefully
you can see a pattern
1 is not in the pattern]

- Claire Grade 6

No because 21 is an odd number, and only even numbers are in the pattern because you add two each time.

2, 4, 6, 8, ...



If I give you a random number, how can you tell if that number will be in the pattern?

2, 4, 6, 8, ...



⚡ If I give you a random number, how can you tell if that number will be in the pattern?

• William Grade 3

if you see the numbers
2, 4, 6, 8, 10, 12 the odd digit
then they are in the pattern

2, 4, 6, 8, ...



⚡ If I give you a random number, how can you tell if that number will be in the pattern?

• Claire Grade 6

I will look at the ^{beginning} pattern and keep doing that
pattern to see if it's even or odd
numbers.

Types of generalization




⚡ **Current – next or recursive**

⚡ Add two each time


you skip count and
see if you land on
that number

I start at 5 and count up 3
every time and see if I pass
that number

Types of generalization 


- Direct
 - $5 \times n$
 - $5 + 3n$

if its a multiple of the number 5

Errors in generalizing 

- Over-generalizing (for the pattern 5, 8, 11, ...)


Divide the number by 3, and if it equals a whole number it is in the pattern.

1, 2, 4, 7, 11... 


What about harder patterns?

- Verbal description alone is appropriate at this level


If you start with a number from the pattern and adding one more each time and see if it would fit.


Patterns from a context 

- ⚡ **Table problem**
 - ⚡ A problem to consider teaching strategies
- ⚡ **Snake problem**
 - ⚡ A problem to really think about patterns

Table Problem 

For a special dinner, St. Patrick's is using square tables that can be put together to make rectangles.

One table can seat 4 people 

Two tables can seat 6 people 

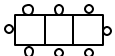



Three tables can seat 8 people 

Table problem 


- ⚡ **How many people can be seated at 4 tables?**
 - ⚡ How could we help a student stuck at this point?
- ⚡ **How many tables would be needed to seat 16 people? How do you know?**
 - ⚡ What are we looking for in an explanation?

Table problem 

- ⚡ **What are some possible numbers of people where there would be seats left over? How do you know?**
- ⚡ This is the first step in generalizing, noticing what is and isn't in the pattern.
- ⚡ What description might a 3rd grader give? A 5th grader?

Table problem 

- ⚡ **Mr. Barrett says that he will tell you how many tables he needs, and then he will ask you how many people are coming. How would you figure out how many people are coming?**
- ⚡ Possible Strategies?


Table problem 





<ul style="list-style-type: none"> ⚡ Current: next or recursive ⚡ Each table adds 2 more people ⚡ The number of people is always even 	<ul style="list-style-type: none"> ⚡ Direct ⚡ 2 + number of tables x 2
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	tables	people
	1	4
	2	6
	3	8
	4	10






Snakes

⚡ Instead of being given a pattern, we are given a starting point and a rule

Starting point: 

Rules:  → 
 → 

Snakes

Pattern	Number of Red Rings	Number of Black Rings	Number of Rings
	1	0	1
	2	1	3
	4	3	7
	8	7	15
	16	15	31

Pattern problems

⚡ The tables and snakes problems are different from the number pattern problems

- ⚡ A series of instances rather than a sequence
- ⚡ Can be modeled
- ⚡ Generalization is within the context of the problem

What does generalizing mean at grades 3–6



- ⌘ **Explaining patterns in sequences of numbers**
 - ⌘ Relating to multiples
 - ⌘ Noting constant or variable change (slope)
 - ⌘ Paying attention to starting points (intercept)
 - ⌘ Identifying which numbers are and aren't in the sequence and why

What does generalizing mean at grades 3–6



- ⌘ **Explaining patterns in context**
 - ⌘ Relate how numbers change to where they come from

 - ⌘ Move fluidly from pattern as numbers to what the pattern means for the context

 - ⌘ Analyze numbers from context as a sequence

 - ⌘ Move pattern beyond context

What to do?



- ⌘ **Talk about sequences, more than just what comes next**
 - ⌘ Relate it to work with multiplication
 - ⌘ Work on a hundreds chart
 - ⌘ Given a rule and a starting number, create the sequence
- ⌘ **Work with pattern problems**
 - ⌘ What's my rule?
 - ⌘ Explore both constant and variable relationships

References



- ⌘ National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
- ⌘ Carpenter, T.P., & Levi, L. (2000) *Developing Conceptions of Algebraic Reasoning in the Primary Grades*. (Report number 00-2). Madison, WI: School of Education, University of Wisconsin-Madison, National Center for Improving Student Learning and Achievement in Mathematics and Science.
- ⌘ Roodhardt, A., Kindt, M., Burrill, G., & Spence, M. (1997). *Patterns and Symbols*. In National Center for Research in Mathematical Sciences Education and Freudenthal Institute (Eds.), *Mathematics in Context: A connected curriculum for Grades 5-8*. Chicago, IL: Encyclopedia Britannica Educational Corporation. ISBN 0030712831
