The teacher-to-teacher initiative was created by the U.S. Department of Education to provide the latest strategies and research on educational practices that work inside a classroom. I'm Linda Plattner and I'm from Strategic Teaching. I'm delighted to be here with you today. The way they taught elementary math or math in the elementary school was significantly different in China than in America. Now you need to write something down – you have to share some of your good thinking. This series features teachers from across the country presenting techniques that can be used with students of all ages. It's just one way the Department of Education is helping teachers get the support they need so no child is left behind.

(MUSIC)

Good morning, I'm Linda Plattner and I'm from Strategic Teaching. I'm delighted to be here with you today. Like many of you, I've spent a fair amount of my career being a math teacher. I've had the privilege of working with students and with adults. I've also spent a fair amount of time helping develop standards, assessments that measure standards, and a lot of different support activities that move standards into the classroom to give teachers the tools that they need. For today we're going to talk about the kind of math that is arithmetic, but often is called basic math. We're going to focus on that for a couple of reasons. Success there is core. It is the foundation for success in all other branches of math. The other thing that's happening across the country as you all know is the push for algebra – that all kids have algebra, and in some states they even need algebra by grade 8. What we're going to do today is talk about the connections between arithmetic and algebra, and how being successful in arithmetic sets kids up to be successful in algebra. I have 3 goals for today. The first is that we deepen your understanding of basic math, of arithmetic. If you'd asked 10 or 15 years ago if I understood arithmetic I would have said yes. If you ask me that today I would be much less confident. The second goal is to become more reflective about how we teach math. Notice I said reflective – not reflexive. For a lot of us – for me, certainly – when the time came to teach something like addition, without thinking I taught the way I learned – and as Carolyn said this morning, that is our natural fallback position – to teach things the way we learned things. The third piece, as I mentioned, is to strengthen that connection. I suspect most of us know the properties of numbers, and may remember the properties of algebra – of basic algebra. What we may not have had the opportunity to do is really connect those two pieces. So, let's get started. Look at these problems – how are they the same? They're all subtraction – good, good, we're getting started here. What else? They all require borrowing. How big are the numbers? 2-digit numbers – okay. A few years ago, a woman named Liping Ma who was born and began teaching in China had a sense that the way they taught elementary math, or math in the elementary school, was significantly different in China than in America, so after she migrated to the U.S., she was at U.C.L.A. working on her doctorate and decided to pose that question. She put together a study of 23 above-average American teachers and she identified those teachers as some that were taking part in a summer training program, and then she selected a sub-group, so 23 above-average American teachers and then she identified 73 typical Chinese teachers. She found – she identified an urban, suburban, and rural school and pulled those Chinese teachers from there. The way she structured the study was to pose a series of questions – math questions – and then look at how each group responded. Can you guess what the first question was? Inaudible. Exactly. The ways of teaching subtraction – and with the American teachers, more than 80% talked about borrowing – that's how I learned – that's how I started out teaching it. A few talked about regrouping and decomposing. With the Chinese teachers – some talked about borrowing, but many more talked about decomposing numbers, regrouping numbers. And many of them talked about doing that in multiple ways. I don't know about you, but I never thought about multiple ways. If we found one way to get the right answer that was good news. I want to take just a minute and let us look at and think about how the Chinese teachers decompose numbers. They didn't do it just for 10's and 1's – they broke the number down into a variety of pieces. So look at the 53. If you think of 53 as 40 plus 13, how does that help you subtract 26? If you've got 40 plus 13 and you've got 26 – you've got 20 plus 6, so you can think of 20 plus 7 or 27. Make sense? What we're going to do now is let you play with this a little bit. So, I want you to take – which problem? – let's look at 62 take away 49. We tried 60 and 2, which we realized wouldn't work with 49, so we got rid of that. 50 and 12, 40 and 22 – we didn't like 40 and 22, because then you have to subtract 22 from 9. Then we started playing with decomposing into 3 different numbers, which we continued over here, so we got 50, 3 and 9 and then 40 and we realized we needed a 3rd number. Zero and 9 – so we weren't sure whether it was harder to teach the zero concept, so that you could have 3-D composition or whether it more sense to keep it fairly simple like this and they'd just have to teach 12 minus 9. Okay, so we looked at it in a very similar way – we just represented it a little bit differently. We tried many different combinations, and I think what we liked was each of our brains seemed to go a different way and the fact that we could share those different ways and then see the other person's way, I think was promoting flexibility in thinking which was kind of fun. So we also have some two-version ways, some three-version ways – but we basically came up with the same ways. Our table came up with several different methods. We started off with 62 minus 49 and we decomposed it the way children might as 60 and 2 and 40 and 9 – subtracted the 40 from the 60 and ended up with 20 and in my classroom children use number lines as well – so they went to the 2 minus 9, which we don't think can happen, but they number line and said the difference between the 2 on the number line and the 9 on the number line is 7, so then they would subtract the leftover ten's and the leftover one's and came up with 13. They also went into decomposing it a different way and said 50 and 20 and 40 and 9 – came up with 10, 3 – added those together – the leftover ten's and the leftover one's to get 13. So we had this particular manner 50 and 10 and 2, then broke up 49 and said 40, 7 and 2 – subtracted in a column – ended up with 13. And then we had the method of using some landmark numbers – something we knew – 49 is close to 50 and that's easier to subtract, so we subtracted 50 from 62 – had 12. And we knew that we had to compensate because 49 is 1 less than 50, so we had taken too much away so we had to add 1 and we still came out with 13. Alright – you guys all deserve a big hand – lots of creativity. Remember I talked about algebra is simply generalized math? In algebra remember seeing lots of things that look like this? And what did you need to do those? You need to factor them – and what is factoring? Decomposing. The more faculty with numbers you can build in children the easier this kind of transition is. So, if you teach kids that 5 apples plus 3 apples is 8 apples, it's easier for them if you help them make that transition that 5B + 3B is 8B. You're looking from the concrete to the abstract. When you do this kind of reaching, like Pete talked about that you talked about – this kind of reasoning with them developing that number sense, in the same way you then can – it's easier to for you to generalize it to algebra. The third piece that's very similar is that the hierarchy of justification is exactly the same in arithmetic and algebra. When you ask kids to explain what they're thinking or prove what they're thinking, the first thing they always say – like, why do you know that's right? How did you get that answer? Mrs. Jones taught me last year – they call in the trump card. The next piece in justification is finding some example problems. I know it's true, because it's true in this case and it's true in this case. The piece we're reaching for is for them to generalize so that they come to the conclusion that for every rectangle the width times the length is the area – not just for this problem – this problem. Here's the second question that Dr. Ma posed. When Dr. Ma asked American teachers almost all of them talked about using manipulatives. What was different was how they used those manipulatives. These manipulatives are plastic silverware. And in order to build that concrete understanding of subtraction, they would have kids take away the 26 by counting and seeing what's left. Do kids need to have that concrete understanding of subtraction? Absolutely. Other teachers in the same way had bundles – so they had 53 that were bundled – I'm not going to get them all out. And in order to take away 26, what did they have to do? Unbundle them – so they would have to build that exchange of one ten is 10 ones. When the Chinese teachers were asked how they would teach, they talked about something called knowledge packets. The Chinese teachers tended to teach by groups of topics – the American teachers tended to teach by little topics – so there was kind of a conceptual piece and the little topics. The teachers each get something called a knowledge packet, although there were slight variations. And if you look – down the center is kind of what we might think of as the scope and sequence – starting with adding and subtracting within 20's, or within 10's, within 20's, larger than 20 and less than 100, and then the whole world of subtracting. The piece that's very different is that they have all of these supporting pieces, so they don't just think about the procedures and concepts needed here, they think about the supporting concepts and procedures that are needed. In order for a student to be successful – even with subtraction – they need to understand the inverse operations. They need to really understand the composition of 10. All of these teachers who aren't circled are those concepts that the Chinese teachers identified as very important – they were the ones that they described as going slow now so that they could go fast later. I tried to create what I thought was how I used to teach subtraction. So I knew that you had subtracting facts – kids had to have that. They needed to know how to subtract without borrowing, and then how to subtract with borrowing. I think the only supporting pieces were I started with addition facts, and I did teach subtraction as an inverse operation. What happens when you do this? You have kind of two problems – you have misinformation implied and misinformation actually taught. People that are willing to give up a Saturday and be here are teachers that really care about their craft, that work very hard to be at the top of their game. Other teachers that haven't had as many opportunities- when I work in the classrooms, I hear them talk about borrowing. When you talk about borrowing, that's like I go to my refrigerator. If I take a gallon of milk from the top shelf and put it in the door, have I borrowed anything? I rearranged things. But when we talk about borrowing it kind of sets up fallacies in kid's heads. Another thing – these are things that I've heard in classroom – well, you just go next door and borrow 10 – no matter what, you just borrow 10. You borrow 10? And is it always 10? Depends on the place value, doesn't it? How about this? – when you subtract, you always put the big number on top. Do you always put the big number on top? In 2nd grade – but what about 7th? Here's another one. When you subtract, you can never have less than zero. When you say these kinds of things, you build up fallacies, misconceptions – and our struggling learners never recover. Somewhere around 5th or 7th grade, they've built all these holes and it's like a house of cards and you start hearing things like, this doesn't make sense – this is stupid. And at some point shortly thereafter it collapses for them and they're done. The next question that Dr. Ma posed was what do you do when you get this instead of this? What is the student struggling with? Place value. Is place value important? Place value is the operating system for numbers. It is the water to fish. When Dr. Ma tabulated the responses, you can see that the American teachers overwhelmingly fell back on some kind of a procedural solution, where the Chinese teachers were much more likely to be conceptually oriented. Another thing that happened with the Chinese teachers is they often took an opportunity to make an extension. This is an example Mr. Chen said – he said this gives me the right answer. I rearranged the products. I wonder how many ways there are to rearrange the products. There are six different ways and there's a couple of ways that you can help kids figure that out. The easiest way – if we look at the first one is A – the second one as B – and the third one as C – then how many numbers could you put in the first place? Three. Sure. And then how many are left over to go in the next place? Two. And what's left? So this is actually the fundamental counting theorem. The step in between for kids would be to list it. You have what? A – you had A, B, C – A, C, B – and continue – so you could list it. My point here is that the Chinese teachers often found ways to make connections. The next problem that Dr. Ma posed was division of fractions. And she asked 3 questions about it – first, can you do the problem? Second – can you do this problem in more than one way? And third, can you build a context. As we often, you mentioned at the very beginning we try to give kids context for it. The one that I often use as an example very similar – I'm taking cookies to the bake sale. I want to make as many as I can. Each batch of cookies is a half a cup, I only have 1 ¾ cups, so how many halves do I have? I have a half, a half, a half, and some left over. If you look here you'll see that only about 40% of American teachers can solve this problem in any way – can solve it. Remember – and I say American teachers – I misspoke. Only 40% of the above-average American teachers in this study could solve this problem. None of them could do it in more than way, and only a few percent could find a story that worked. Now these are our teachers that are working hard – using all the skills they have at their disposal. One of the things that's really interesting to me is the way American teachers are prepared and the Chinese teachers are prepared. Elementary Chinese teachers only go to school through grade 9 – and then they spend a couple of years preparing to be teachers. When they're assigned, they have no area of specialty. When they're assigned to a school, when they get there, the school says, hmmm, we need a social studies teacher, or hmmm we need a math teacher. What they have, though, is a really deep, profound understanding of the math they know. The way they learned is the way they were taught. Now in America, like when I was working on my Master's in math, I took classes like proving the fundamental theorem of calculus, so that when you integrate a differential you end up with the same place. Now that's not helpful when you teach elementary – so teachers are doing the best they can with the tools they've been given. In her book, Dr. Ma also has a geometry problem that we just didn't have time to look at today, but you can see that place value holds together subtraction and multiplication. It's core to both of those. The concept of inverse operations – that ties together the subtraction and division that's how we teach those two right? Even here, that's how we teach those two. Now the meaning of addition – that ties to multiplication and it ties to geometry. We do some work with geometry in what? Using – visual models, arrays – exactly. And then the media of multiplication cuts across all three. Have you thought about those pieces before – how those things tie together? These are the things that we want kids to understand from basic arithmetic – basic math – that sets them up for success. They need to understand the fundamental properties of numbers. How one acts – how zero acts - communitive – associative – distributive. These are pieces that can all be taught using concrete manipulatives and moving exactly in the same way that Leslie talked about on to the abstract. They need to understand that a sign represents a relationship. So many children believe that when you see the equal sign it means write the answer. I have a colleague that said I'm going to know that I'm doing my job when I say some time what is 6 plus 4 and a student says 7 plus 3. The equal sign is a relationship. And the last thing is that operations represent relationships. 4B is a relationship.

Americans talk about basic math – Chinese talk about fundamental math. It's foundational – not basic – it's just fundamental. I know I've said this enough, but how children are taught elementary math sets them up for success with algebra. And the last piece – the knowledge gap between American and Chinese teachers parallels the learning gap between American and Chinese students. There are three follow-up activities. One is that one we didn't get to today but would be great to do with colleagues. If you sat down with them and tried to create a learning packet – a knowledge packet – for multiplication – think about the core skills and concepts – think about the supporting skills and concepts. There is an algorithm for dividing – you can look at that yourself and ask yourself these questions. Is it procedural or conceptual? When I use it how can I improve it? And then the last piece, which I think is the most important. Take at least one lesson from your text. Work with your colleagues to identify the pieces of algebra that are there – maybe buried. Strengthen them – make them explicit. Okay – you guys have been awesome. Go forth – learn more.

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