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The Teacher-to-Teacher initiative was created by the US Department of Education to provide the latest strategies and research on educational practices that work in the classroom. This series features teachers from across the country presenting techniques that can be used with students of all ages. It's just one way the department of education is helping teachers get the support they need so that no child is left behind. Hi, I'm Beth Cole and I'm a sixth grade math teacher at St. Patrick's Episcopal Day School in Washington, DC. Our school is an Episcopal School, we have students from ages three years old through eighth graders. We're contained on two campuses, one campus for our nursery and elementary school, and a second campus for our seventh and eighth grades. Good afternoon, my name is Beth Cole, they asked if I could introduce myself and I figured I probably could since I've known me all my life, so I guess that will work. [laughter] I'm just going to tell you a couple brief things. Teacher at St. Patrick's Episcopal Day School in Washington, DC, which means I have the opportunity not to teach in the real world, I teach students with highly motivated parents. That doesn't necessarily mean I have highly motivated students, and our school's claim to fame currently is that the Edwards' children go to my school. So next year is going to be an exciting year for us. We have three year olds up to eighth graders in our school, so I get to see a wide variety, and I'm the sixth grade math teacher. That's who I am.

And the most important thing I think about me is the fact that I'm the daughter of two math teachers so when I was growing up there were two things that I was very clear about. The first one was that I was not going to be math teacher, and the second was that real families did not talk about math at the dinner table, and did not spend their time thinking about math and worrying about how best to teach math and so forth. As you can tell I failed miserably in both of those things that I knew were true about myself, because I'm a math teacher and in fact I've spent a lot of time thinking and worrying about various aspects of mathematics education. My first teaching job was teaching seventh and eighth graders. The eighth graders, some of whom I taught a course called Algebra I.

That was very clear, I knew what I was to teach, there was a book, I was to start at the beginning, I was to get most of the way through the book by the end of the year. Now the seventh graders and some of the eighth graders I taught a course called pre-Algebra. That was much less clear.

There was this book, it was older than I was. It was really, really boring, and it was pretty much arithmetic. And I knew these kids knew arithmetic, they just didn't have the kinds of reasoning skills for placing them in an Algebra I course and the hardest part was the students who I taught pre-Algebra to in seventh grade and then I found the next year I was to teach pre-Algebra to them again.

[laughter] They were good kids, they were just not abstract thinkers.

They weren't ready to be in an Algebra I course, but there were lots of mathematics that they were ready to do. So I spent much of my time there asking my colleagues what they thought it meant to get kids ready to take Algebra and they would humor me and answer my questions and talk about it at

dinner. It was a boarding school so we all ate together, and then finally a couple of them starting saying things like, "Beth, you know when people ask questions like this they go to graduate school, and maybe you should go to graduate school." And so I did and I had the wonderful opportunity there to work on a curriculum development project and work with amazing people who knew a lot about the kinds of experiences that middle school students needed to have in order to be ready for a formal Algebra course and I feel pretty confident now that I know some of the things that need to happen in sixth grade, seventh grade, eighth grade in order for these students to be ready.

I left graduate school and I decided that in fact I wanted to go back into the real world, such as it is, and I decided to go back into teaching in the classroom.

And I found a job in an elementary school. So now I knew what it meant for middle schoolers to get ready for formal Algebra but I really wasn't sure what it meant for elementary students and in particular the grades over which I have the most influence which are grades three through six. What kinds of experiences do these students need to have so that they're ready for the kinds of things that I now believe are critical for middle schoolers to have to be ready for a formal Algebra course. And it turns out that we know a lot about Algebra in the elementary school. In fact there was a study conducted by Turk and Tufts University which identified seven aspects of Algebra that should really be and can really be taught in the elementary school. Now, I don't have enough time to tell you about seven aspects, so rest assured that's not going to happen.

So I decided to focus and everything that I read kept talking about patterns.

Patterns, patterns, patterns.... This is where everybody seems to agree is one of the primary places to focus in the elementary school to prepare students for Algebra study. So I did what any good teacher of the century does when faced with wondering about how to talk about something, I went to Google. [laughter]

And I found some very nice quotes. All life is patterned, but we can't always see the pattern when we're part of it. In fact what I'm going to talk about today is helping our students see the pattern and what it means to see a pattern.

Art is the imposing of a pattern on experience and our aesthetic enjoyment is recognition of the pattern and I contend that's what makes the mathematics powerful. Is recognizing the pattern, really seeing and understanding what's there. And then I was listening through my wall of my classroom, do you all do that? It's really quiet in your room, and so you kind of put your ear.... And I heard one of the lovely students who I will teach in September say, "When will we be done with patterns and go back to Math?" [laughter] So, there we go.

That's why I started with patterns. Now suddenly I had a thought, see I spent my whole career worrying about Algebra and you know what should we do, and different grade levels, but is my effort misplaced? Is there something else that I should be worrying about? Well no. We know the disappointing results on international comparisons and one of the things that they particularly point to is our students, the US students' difficulty with Algebra. And so I started to look at why some people think this is the case and we know that the transition from arithmetic to Algebra is not an easy one, that's where we lose an awful lot of kids. And in fact two researchers, Carpenter and Levy, propose that it's that

separation, the fact that you take arithmetic for a certain amount of time, and then you stop doing that and then you take Algebra and it's that separation that has really caused a lot of the difficulty that makes it hard for kids to learn Algebra. So we really do have a need to focus on Algebra at the early years.

So we need to focus, but this doesn't mean teaching Algebra I to third graders.

Now, I don't know where you all took Algebra I, and I won't tell you where I took Algebra I, but the fact of the matter is that the course that I took that was called Algebra I, I could teach to third graders. It was all about formulas, right?

If the problem looked like this, then you do this. If the problem looks like this, then you do this. Okay? It wasn't a lot about thinking about Algebra, it was a lot about doing. Luckily that's no longer how we teach Algebra. And so what we need to be doing is really focusing on some of these Algebraic ideas so that when our students get to the Algebraic thinking level then are able to think abstractly, they're ready. And really it comes down to two big things. Making generalizations and using symbols. And today I'm mostly going to focus on the idea of making generalizations. We're going to use a few symbols because when you make a generalization you use a symbol, but really I'm going to talk today about generalizations. And in fact one of the easiest ways to think about generalization is to think about generalizing patterns and the NCTM calls us at all grade levels to have students understand patterns, relations, and functions. So certainly this is where we need to be focusing or one of the many places where we need to be focusing our time and effort. So that about patterns. Now one of the real privileges of teaching in a school that has itty bitty children in it is I can go down there and find out what they're doing. So I walked down stairs one day and I said, "So, tell me about patterns in kindergarten" and in fact they were doing patterns that day because they do a lot with patterns in kindergarten. And this is what they had up, the teacher had made beautiful circles, cut out of construction paper and laminated with magnets on the back, which convinced me that I can't teach kindergarten, but she had put up this nice pattern and when it was each child's turn they would come up and they would pick up whichever one was to go next and put it on and they were continuing a pattern and building this across the blackboard in their classroom. And it was very easy for some of the kindergartners and it was more difficult for others and their patterns varied, and the teacher, depending on who was coming up next might change the pattern a little bit, and it was a fantastic exercise for kindergartners. The troubling part is when we see third graders being asked to do essentially the same thing, just continue the pattern. So what I'm going to talk about today is by the time they're in third grade we've got to be asking them to do more, and that more is making the steps towards generalization. And we can see that progression in the words that NCTM used when they developed the standards. We go from recognize, describe, and extend to describe, extend, and make generalizations about. By six eight we're representing, analyzing and generalizing. And you can imagine how that's moved further as we move into the high school years. So, enough listening to me. Here are some patterns, and if you look in your hand outs, now this is tab number seven and you have to flip through the pages that have my

slides on them and hopefully, eventually you will find a sheet that looks remarkably like this.

And this has three questions on it, I'm going to start by reading the questions and then what I would like for you to do is answer each of those three questions for each of these four patterns, and you've got to work in groups and you've got to talk to each other because I teach sixth graders and when it gets quiet I get nervous, because it's never quiet in a sixth grade classroom. So the questions you're going to answer are: What are the next three numbers in this pattern and how do you know? Would 43 ever be in the pattern, how do you know? And if I give you a random number, how can you tell if that number will be in the pattern? Everybody understand what we're doing, answering three questions for each of four patterns. All right. Talk to each other. [various voices] The pattern is even numbers. Right. How do you know? How do you know they're even numbers? Will 43 ever be in the pattern? No. Because 43 is odd. 5, 10, 15, 20 would be 25, 30, 35. They're patterning by five. And how we know, patterning by five. So, 25, 30, 35. And then 5, 8, 11, and 14 is increased by 3. Okay, the next three are 25, 30, and 35. How do we know this? We're counting by fives. Write the difference between the previous numbers in the pattern. They increase by five. And counting by fives, that's what kids say, right? They know this pattern, it's the counting by fives. The next one gets a little more interesting. Yes, it was a little harder for me, it took me a bit to come up with this one. (laughter) Not counting by threes. Well it's adding three, I don't think it's counting by threes, it's adding three. Did you get it? Um hmm. Did you get the next three numbers? Yes, I could do that. I don't know where to go from there. Trying to get the slope of this line so that if that was two, and that was five and over one, up three, so maybe it's  $3n...$  Alright, shall we start talking about these a little bit? Everybody either done or annoyed? You didn't get the last one? Alright, so I made up these patterns and I made up the questions but the questions are a particular set of questions in a particular order. Right? They start with something that everyone can do, starting out with just giving the next couple of numbers, give kids a chance to explain how they're getting those numbers, because that's the first part of generalizing.

Can you explain how you figured out what came next? The next question is the second step in generalizing. What is or isn't a part of the pattern and how can I tell? The third one really is the push to generalization. I'm not going to tell you what number I'm going to give you, but can you figure out if it's going to be there. So I gave this set of questions to some third graders and to some sixth graders. Let's look at what I found. The first question was what are the next three numbers in the pattern, how do you know? And this was for 2, 4, 6, 8.

Now a third grader I know said, "it is counting by twos, the next three numbers are 10, 12, and 14." Alright, he's very clear, it's a pattern he's very familiar with and he's able to explain exactly what he did. He knows the counting by twos, he's used them before, he identifies this as a pattern he knows and understands. A sixth grader says, "10, 12, 14, the pattern is plus 2 each time."

So she's doing the same kind of thing, in this case she's giving us a rule, alright. How do we get from one to the next? We add two each time. Alright?

Now I asked them about 21 and I asked you all about 43. Alright, but would 21 ever be in the pattern, how do you know? William says, "No, if you look carefully you can see a pattern, one is not in the pattern." What the heck is William talking about? What is he looking at? The one's place, right, yeah, he's looking at the ones digit. Again, he's connecting it to the fact that he already knows about this pattern. He knows something about the counting by twos.

And he knows something about what they end in and one is not one of the things they end in. Claire says, "No, because 21 is an odd number and only even numbers are in the pattern because you add 2 each time." She's really close to a generalization there, right? But is there any way that you can get an odd number by adding two each time? Yeah, if you start with an odd number, okay. So we would have liked to have Claire tell us no, it's always even because you add two each time starting from an even number, right.

But she's definitely on the right track, she's working towards a generalization here. If I give you a random number how can you tell if that number will be in the pattern. Now a lot of third graders did this by giving me which random number I should pick. [laughter] So to a lot of third graders the idea of not knowing what that number was going to be was beyond what they were able to grapple with, and reasonably so, I mean this is a hard question, an abstract question. But William took up the challenge and he said, "If you see the numbers 2, 4, 6, 8, 10 in the one's digit, then they are in the pattern."

Okay. Now I've never seen 10 in a one's digit, but we're darn close here, right, I mean he really knows what he's talking about, he was able to verbalize it much more clearly in this answer than in the previous one, right. So I think he thought, "Oh, she's serious about this, she keeps on asking me the same thing, I better get clear." And the sixth grader said, "I will look at the beginning pattern and keep doing that pattern and see and look at even and odd numbers." You've taught sixth graders, right, this is how they talk. [laughter]

Right? But she's getting there, right, not written necessarily the way we want it so we could have a language arts teacher kind of work on her. [laughter]

But you can see, now she realizes that you have to, not only keep doing the same thing each time, but really start with what you're supposed to start with and keep doing the same each time and then she throws in that stuff about even and odd also. Right? She got a bunch of stuff mixed up in there, you can tell she's on the right track. Now I'm not going to go through each of the patterns for the student work, but what I do want to share with you is some general things that students did. Students tended to generalize in one of two ways, and really these are the two primary ways to generalize. One way is the current next, or the recursive way. Hey, I like current next, whatever you have currently, this is what you do to get the next one. Okay. My husband the computer scientist tells me that's recursive. Alright, so they're two different words, mean the same thing. How do you get from one item in the list to the next item in the list, and one way you might do that is add two each time.

Or as students wrote, you skip count and see if you land on that number.

Right? Skip counting is a very powerful tool that we use in our primary grades now and in fact some of my third graders were able to very clearly show me

whether something was in or out of the pattern by using the skip counting on the hundreds chart. So they're able to mark on their hundreds chart and show me how skip counting works and how a particular number would or would not be in the pattern. A sixth grader wrote, I would start at five and count up three every time and see if I pass that number. So that was for the third one, right.

They tell us where they're going to start and they tell us exactly how to create the pattern. I'm going to add three each time, see if I pass it.

You can also write a direct formula, right. Each of these is five times something. Or each of these is five plus three times something.

So if I want the twelfth item on the list I can just do five times twelve.

Or I want the 354<sup>th</sup> item in the list I can do five plus three times 354.

Okay. And a student carries that out if it is a multiple of the number five.

That's really the same thing. Now the nice thing about students is in addition to having nice correct answers we also get consistent incorrect answers, right, things that lots of kids do and in fact on this the main thing that I saw was over generalizing. They saw the pattern that they wanted to see.

So for the pattern 5, 8, 11. The student wrote divide the number by three and if it equals a whole number then it can be in the pattern. Close but totally wrong.

Right? They saw the skip by three, they knew that had to do with multiples of three, so this child knows a lot about patterns. They know that if you go up by the same number each time that's a multiplication that you can think of it that way, it was the starting not at 1 or starting not at 3 that messed them up.

But they were on the right track. So what about harder patterns?

All right, some of you are muttering. 1, 2, 4, 7.

How did you all describe that pattern? How do you create that pattern?

Somebody tell me what they said? (inaudible) Some of you did it. It's a numerical order. How do you figure out how you're going to get to the next one?

You guys did it. Yeah, can you try and talk through it?

We added one to the previous number we added.

Good. You got it.

You added consecutive numbers would be one way to say it, or what you added kept increasing, or the students said, if you start with a number from the pattern, that's good, we got starting at the pattern.

And adding one more each time, so that's that piece that's not quite clear, but they're getting at it, and see if it would fit.

That's completely appropriate.

I want kids to see patterns like this, but I don't need them to be able to write a formula yet.

If they can do that in sixth grade there's nothing left, right?

We've got to leave something for the middle school and high school teachers to teach otherwise they'd be bored.

Alright, now in case you were worried, there is a formula that gives you this.

Want to know it? Okay. It's one half  $N$  squared minus one half  $N$  plus 1.

Now don't you feel smarter for knowing that? [laughter] It's one half  $N$  squared minus one half  $N$  plus 1. Truth be told I had to call my parents. [laughter] Okay,

I knew some stuff about it and I couldn't make it work out right and it was 10:00 at night and I just went, "Dad!" He's like, "You're too old to do this."

So I hope that these couple of examples have made you start to think about what we can do with patterns, purely numerical patterns with kids. Okay. Because we see these purely numerical patterns all the time. Now one of the advantages of the Texas TAKS Test is that they have lots of stuff on line, they have lots release items that those of us that do presentations can go look at.

So I went and I just looked a little bit and in fact I found at every grade level, from three through six because those are all the ones I looked at, had pattern problems. Okay, many of them were give the next number, pick from the list which of these is the next number in the pattern, okay, or which of these rules would create the next number in the pattern. Same kinds of things we're talking about here but I hope you can see the power if you ask kids to struggle with why, how easy it is if then they just have to pick out the next one. So while we're working with these patterns and we're preparing them for formal Algebra and we're getting them to think about what generalization means, we're making some of the stuff that shows up on the standardized test really easy for them.

So all that set of problems all were just here's a list of numbers. Now we're going to look at some patterns that come from context. Infinitely more interesting in my opinion. The first one is the table problem and we're going to use that to talk about some teaching strategies. How to help kids see these patterns, and the next one is the snake problem which is a chance to really think about some pretty cool patterns. So if you look at the next page in your handout you should be able to find the table problem and it says for a special dinner St. Patrick's is using square tables that can be put together to make rectangles. Before you start we're not putting them at angles, no, just put them together to make long rectangles, okay. Can you tell I teach middle school?

One table can seat four people, two tables can seat six people, three tables can seat eight people, and if you go in that handout then there are a series of questions about these tables. So if you could please work at your tables on the table problem. One table can seat four, two tables six. So we're adding two each time, right? Four tables is ten. So it should be seven for sixteen.

I just started at four because I knew that one table seated four and then I skip counted by twos. Right, right. Draw it out. I had to do that the other day, we were doing a problem..... [music]

[music]

...because we're actually taking away two. If you think about that, it has to be something minus two as well. So let's talk about what teachers love to talk about. How are we going to help kids? So we give this problem, we ask the first question, how many people can be seated at four tables, you've got all these groups and they're working great and you've got one group and they're all staring at each other. And it's not because they're not trying, it's because they're stuck. Right? What are some strategies we can use right at the beginning to help a student who's stuck? Some of you said it. Draw a picture. Right? This problem lends itself very nicely to drawing pictures. It's really easy to draw the pictures, right? You need to be able to draw something that's kind of square and something that kind of looks like the top of somebody's head, right? Most of us can draw this. Now we have students in our classrooms who can't or for whom that is so physically cumbersome that they're going to forget what they're drawing by the time they draw it. What do we do for those kids? (inaudible) Good, some kind of graph paper probably with big squares, right, where they can mark them off. Okay, what else? Tiles. Great, tiles and counters, right. Or I've used post-its and counters, right, when you can't find the tiles or post-its and paperclips, you know. Index cards, right, index cards cut in half are perfect, right? So see how it's really easy to get kids to model this and so that's one of the key ideas is that we really want to have patterns where everybody has access to beginning to build the pattern. Maybe not everybody in our classroom can get to the final generalization, but absolutely everyone can understand where this pattern started and how it grows. So how many tables would be needed to seat 16 people? How do you know? Okay, we've got kids, they've got four tables, and they're willing to use the post-it notes and the paper clips but we want to push them a little bit. What way can we push kids to get to this problem, where we're not getting them to a formula or to an abstraction right away, but we want them to move away a little bit from the physical manipulatives. What might we ask a child to do? Some of you did it, I saw it.

There we go, make a table. A table about the tables, right. [laughter] So we can make a table that says something like, number of tables, number of smiling faces, right. If we have one table we have four. Two? And pretty quickly kids can continue that till they get to 16. Right. They're not going to have to miss recess, right? It's not that long to do that kind of table. And the act of making the table this way might make the patterns a little bit clearer to some of the children. So what are some possible numbers of people where there would be seats left over? This is really where we start making that step to generalization and one of the easiest ways to begin that step is to say, okay, we've got to figure out what's not in the pattern. Because once we know what's not there, it's easier to tell if we've identified the things that are there, okay?

And so pretty quickly what did you see? What are some numbers that are never going to show up in a list like this? Odd. Odd numbers, right? In this case that's nice. Some kids might tell you odd numbers and what even number?

Two. Right. Because if we have one table we've already got four people taken care of. And so the third grader tends to say something like, "Odd numbers don't work", we hope by fifth or sixth grade they might say, "Odd numbers don't work and also two doesn't work" or they might say, "Any number that is not a multiple of two, other than two" or something like that. So Mr. Barrett says he will tell you how many tables he needs and then ask you how many people are coming. So you don't know, all he's going to say is, "Set up 523 tables". We have a really really big gym... No, I don't know. [laughter] And he wants to know how many people. So how can we figure that out? Alright, we could keep going forever and we we've all taught students who are willing to do this for a really long time, okay. But that's what we're trying to move beyond. So what kinds of things might a student say at this point? What kind of generalization might they make? You guys are so proud of yours back there, what was it?

A formula. A formula and what was your formula?  $2n$  plus 2 where  $n$  is the number of places. Okay. I'm going to write it a little differently, I'm just going to write 2 times the number of tables, plus 2. Why does that formula work?

Not good enough to just have a formula, I think the whole point of doing things contextually is that now you have a context to describe your formula. She's explaining it over there. Tell me, go ahead. [laughter] She had good hand motions. Well with this picture that I drew. Yeah, draw a little picture. Okay so with our picture of all of our rectangles, you know you've always got a person on each side of that table, so there's your two at the table, so there's two times the number of tables, a person on this side and that side, and then on each end that should plus two, you've always got a person at the head of the table and at the end of the table, that's the plus two. Okay, and you see how the context gives the power to the formula? Right? It's not just a formula that explains what you do with this number to get to this number, although it is a formula that does that, right? It explains the physical situation that we're talking about here.

Did anybody write their formula differently? A  $T$  for tables, right, and I'm not doing the whole symbols talk now, so...[laughter] It was interesting. When I did this in Pittsburgh somebody said, "Oh, my formula was different. My formula was the number of tables plus 1 times 2. Does it work? Three plus one is four times two is eight. Four plus one is five times two is ten.

Looking good right, looks like it works. How can we explain that? Well it works because you're adding, your plus one and you multiply it by two. Okay, so we could use the distributive property, right? But we're in fifth grade and we're not talking about the distributive property yet, so you're right, you're absolutely right, how physically can we describe that? What's  $T$  plus 1, what people are we talking about when we change colors here?  $T$  plus 1, where are those people?

Nope. Good. This person, this person, this person, this person, and one end, right. Then we're going to double all these people, right? This person doubles down here, this doubles down there, this person doubles down there, this person doubles down there, and this one goes way over here. Okay? So the neat thing about working with patterns, and in particular working with generalizations in context is the fact that all the patterns relate to the context.

So even if as a student I wasn't able to come up with this or this, I can now understand why those make sense. It's not just random symbols floating around on the blackboard. (inaudible) Right. The next one is plus four. Then plus five. Yes, it's definitely there, right. So one of the neat things about any pattern problem is that as you keep working with it there are more and more patterns. So she's talking about, there's also an additive, we talked about a multiplicative relationship, essentially between this and this, right? But there's also an increasingly additive, and you guys were doing it as subtraction, right?

You all were talking about the difference was three, the difference was four, the difference was five, okay. And you can explain that in terms of the number of tables, let me think, as you add another table you've got to think of it as starting with the number of tables and replacing all the number of people, I'm not going to be able to do it on my feet, but that's how it works, okay? And the two kinds of generalization that I talked about before come up in this right? The current next to the recursive, each table adds two more people. Every time you stick another table in there you can seat two more people. And the number of people is always even, right, that's what we talked about. And we can have a direct formula, I wrote it a little differently, two plus the number of tables times two, or just the reverse of what I wrote because I like to put the people on the ends first because I know who they're going to be, right, and then you fill in the people.

And then the idea that you can make a table to show the relationship, and once you do that you can see that the current next, or the recursive gives you a pattern going down and a direct gives you a pattern going across. Okay, so that's where those two different kinds of generalizations come to play. So, I want to move on to a different problem. This problem is about snakes. As I tell my students, these are genuine fictitious snakes, okay, they do not exist in real life, they're completely made up because they have really, really nifty patterns, okay.

And in order to experience these patterns we're going to have a few toys here and everybody needs to have in front of them one red chip. Okay, so whenever a snake is born, the snake is born with one red ring. And these snakes grow in very interesting ways because as the snake grows that red ring splits and a black ring grows in-between. So after the snake has grown one step, right, I don't even want to think about how long it takes for this to happen, right?

After the snake has grown one step, instead of being just one red ring, the snake has a red ring, a black ring, a red ring. So one red ring splits and becomes a red, a black, and a red. Now that snake is at stage two.

Now that same pattern continues. So everybody's got a red, black, red, except black is yellow. There you go. Now that same pattern is going to continue.

That very first red ring is going to split. So instead of just being a red ring it's going to become red, black, red. The black ring is going to stay the same, and then that final red ring is going to become red, black, red. It's tricky, and it doesn't necessarily grow the way you think it's going to grow. So why don't you all get started, there's a description, a verbal description of this in the handout, and I'm going to come around to each table and make sure that you have it growing correctly. Okay, as you begin to look at those questions. You got it?

A ring develops into the middle. The next thing is, each black ring stays the same. The process continues in the same way as the snake grows. Okay, this would be step one, and this would be step two, right? This one drops down. These two split. And this comes down. So I want you to look at this table and then there are a variety of questions that follow it. I'll count them up. But this was... It's a neat pattern. Good. The key is making sure they understand how to build it to begin with and that tends to have to be pretty hands on. I mean just like I did, going around make sure everybody has it building right. So let's talk about some of the patterns that are here. First let's fill in my missing numbers. I'm not doing all the r's and b's, I made them they're lovely, they're over there. Okay. If there are eight red, how many black are there? Seven. And how many total? Fifteen. If there are fifteen black, how many red are there? Sixteen, and how many total. Okay, so start talking to me about the patterns in this table. Give me one. If you give me the first one you get to choose. The double, tell me more, what doubles. Okay, the red double each time. Good, the red double each time, why? Who can explain to me what about these snakes makes it so that the red doubles each time? Because they break in half. Good, because they break in half, right. Every one red becomes two reds, that's pretty much the definition of doubling, right? Somebody want to give me another pattern that they see? Good. The black is always one less than the red. Okay. Why does that make sense? Because for each layer you only get one black. Kind of, yeah, you're on the right track. That's true, but that explains a different pattern, that doesn't tell us why it's one less. Because the red produces two reds, it splits in half and adds black. You guys are explaining a good pattern, but you're not explaining why it's always one less. Think about pairing them all up together, a red with a black, what do you have? You have a red left over at the end. Okay, so you must have one fewer black. Other patterns that people saw? The difference, as your working your way down, is always doubling. Okay, so the difference is doubling. Here the difference is two, four, eight, sixteen. Okay, that relates to something else. Did anybody see it? I'll help. Right, it always adds across, that's reassuring, right that both parts add up to the total. Look at what I'm pointing to. Oh! Good. The total from one is the number of blacks the next time, that's what you guys were describing. Right, that's what you're telling me, right. Because every time the red splits every red makes a black and every black makes a black, right. So if I started out and I had two red and one black. These two reds made a black and this one black made a black so that gave me my three blacks the next time. Let me say that again because we got a little dazed look from the room. Okay? I have this one. It goes red, black, red. This red produces a black. This black produces a black and this red produces a black. So, every chip here, is responsible for a black chip down here. That's why this number is the same as that number. Yes? Oh then why does the difference double? Well the difference doubling is related to the red. Oh I thought that's what we were talking about. Sorry. Okay, so we explain this one, and then going along with that is that this difference doubles because those numbers are doubling. And what this difference is, is what these are going up by. Anybody see any other patterns in there? There probably are some. So I

hope you can see the kind of richness that can come out of this little pattern. It's not trivial to explain how the pattern grows, but it's relatively easy. I think most of you can visualize explaining this to most of the students that you teach. Manipulatives are key to building this and understanding this, but their cheap. Little scraps of paper will work to build these. Then you just have to be careful that somebody doesn't go rushing out of the room and blow everybody's snakes off their desks. Other than that... So the tables and the snakes problems are different from the number problem patterns. The ones we did at the beginning and then these were different. In this case, it's a series of instances, rather than a sequence. Right? We had the first snake, the second snake, the third snake, right? They can be modeled. Right? We built the table, or we drew the tables. We built the snakes. And the generalization, we can tie back to the context of the problem. We can make sense of the fact that it's a doubling pattern, because we can see that every one red becomes two. That gives us a way to understand the pattern. That doesn't exist if the pattern is just a bunch of numbers in a row. And the other piece is that both of those show a more functional relationship. So what does this mean? When you go back, what do I want, what changes would I like to see? I think it's important that we look at just patterns of numbers, right, but that we do more than just continue the pattern. Our primary colleagues are doing a great job of that. It's our turn to take the next step. We spend a lot of time in the upper elementary grades worrying about multiplication. Multiplication is great patterns. Right? It is a pattern. We want to talk about constant change and we want to talk about variable change. It goes up by two each time. It goes up by two, it goes up by three, it goes up by four. If it only ever does the same thing, we're really only talking to kids about lines. Right? I didn't even bring in the whole graphing part of this, but if we think about it, a constant change is a slope. A variable change, now we're talking about a curve. And if we think about those differences, we're really talking about calculus, but don't tell anybody because we're in the elementary school. [laughter] We've got to pay attention to the starting points and we've got to help our kids understand that the starting point is important. And that different starting points make a difference. And it turns out that that starting point is the intercept, depending on how you define your equation. And we always want to be asking kids about what numbers are in the sequence and what numbers aren't in the sequence, and why? The why is the difference, right? If we don't ask them why, we're just having them continue the pattern. So that's what we need to be doing. When we work on patterns in context, we always want to talk about how the numbers change and why they change. Where do they come from? Why does it double? Why is it one less? Why does this make sense? That's why you work in a context, because then you have a context to help you explain things. We want kids to move back and forth. We want them to see the pattern as numbers. As soon as you started to see this, you could see that it was doubling. Because that's a pattern you recognize. We want kids to know that and recognize that and then step back to the context, and say "okay, it's a pattern I recognize, how can I make sense out of it?" We want them to be about to work with these numbers as a sequence of numbers, just as we want them to go back. And the neat thing about context and

numerical parts of it is you can move beyond the context. I don't ever want to see a snake that's 499 rings long, but we can talk about that one. Let's talk about that one for a second. The question was, could you ever have a snake where the total number of rings is 499? We got one vote for yes, we got one vote for no, sounds like it's a typical classroom. What do you think? Well, all these numbers are odd right? And 499 is odd. So far so good. What do we know that needs to be true about the red and the black? (inaudible) ...and they have to be one apart. Right? So, for 499, I can probably come up with this, it would have to be what? 250 and 249? Did I subtract correctly? So we have 250 and 249, give us 499. Well it looks good to me, it's even, it's odd, she's shaking her head, talk to me. It would be 16 times two, times two, times two... Or we can go backwards, right? What would have to come right before it? If 250 was here, what would come right before it? 125. Do you see a problem? It's suddenly odd. Okay? So, I hope you can see, we were able to play with this pattern as numbers. There's no way I'm laying out 499 little pieces of paper, right? Talk about a nightmare. So, we move the pattern beyond context. That's the cool part about math. Right? That's the generalization, that we start with a context, we start with something we build and something that makes sense, and we move beyond it to understand something more about the numbers and the world. So what are we going to do? You've got to talk about sequences and you've got to talk about more than just what comes next. Relate it to the work with multiplication, your work on the hundreds chart. A neat activity is to give a rule and a starting number and have the kids create their sequence. That's perfect, they're coming in from recess and you've got to get them seated and less sweaty. I don't know about yours, but we have to get them less sweaty. So give them a rule, you have a rule on the board, as each kid walks in, you give them a starting number. Say, "Write me my sequence." You can extend that, I give you one number, I give you a different number. How can you tell if you're going to have the same sequence or not? Work with pattern problems. "What's my rule?" is a great problem. They yell out a number, you yell a number back. They have to figure out what you're doing each time. I'll date myself, when I was in fifth grade, we had this thing in our classroom, it looked like R2D2. I was very excited. You took a stack of cards that were all the same color, and they had a number on the top and you slid it in and it came out with a different number on the bottom. It took me like 3 weeks to figure out it just turned the card over. But it was a "What's my rule?" game. Right? And I loved that game. This was when I was still in denial about being a math teacher, you understand? But, it was fun, it was a puzzle, and that's a neat way to think about getting kids to think about these kinds of rules. Where the pattern doesn't have to be sequential. And again, I'll give my little pitch for both constant and variable relationships. Okay? It's fun to look at these things that are constant. It's important that our kids see stuff that isn't. Otherwise they're going to think everything is. And in the real world, when we start to talk about modeling real phenomenon, very little works in a line. So we really want them to see differences and different kinds of change. So what do you do? Try this. Okay? There are activities that follow-up this session. And those are going to be available for you to try with your students

and questions available to think about what happened when I did this. And that's the end. I did some quotes, so those are the references to prove I didn't make them up and I'd be happy to take any questions that anybody has... [music] For more information or a free online follow-up to this program, log on to [www.ed.gov/teacherinitiative](http://www.ed.gov/teacherinitiative). This broadcast and the follow-up are brought to you through a partnership of the U.S. Department of Education and the Panhandle Area Educational Consortium.