Mathematics

PreK -12
Sunshine State Standards and Instructional Practices

number sense
number concepts
number operations
measurement
geometry
spatial sense
algebraic thinking
data analysis
probability

A guide for teachers to help students achieve the Sunshine State Standards
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Introduction

CHAPTER HIGHLIGHTS
• The Need for Reform
• The Value of a Framework
• The Standards Movement
• Standards Initiatives in Mathematics
• How Was This Framework Developed?

The Need for Reform

All over this country, educators, citizens, and political and business leaders are working toward education reform. An increasingly service-oriented, information-based society that is virtually exploding with expanding knowledge demands that everyone have the opportunity to acquire the necessary skills to succeed in the information age. Reform is needed to keep pace with opportunities presented by technological advances, new knowledge about how students learn, and new ideas about how people can improve the productivity and quality of their organizations. The need for schools to change is reinforced by the recognition that teaching and learning are most effective when the diverse needs of students are met. Worldwide economic changes and an array of political and social issues also call for new ways of operating schools.

These new conditions require citizens who are prepared to make well-reasoned, thoughtful, and healthy lifelong decisions in an ever-changing world. Students must learn how to locate, comprehend, interpret, evaluate, manage, and apply information from a variety of sources and media. They must learn how to communicate effectively in a variety of settings and for a variety of purposes through many different media. They must develop mathematical skills to analyze information, solve problems, and create products to meet new needs. They must become creative and critical thinkers, skilled in systematic problem solving. They must learn to wisely allocate resources.
used to solve problems. They must learn to understand systems and to use technology. They must develop the integrity to work cooperatively and effectively with people from many diverse backgrounds.

Florida has created a school improvement and accountability initiative to reform education in its public schools. The goal of this initiative is to raise student achievement to world-class levels. To this end, new, high-level academic standards, called the Sunshine State Standards, have been created delineating expected achievement by all students. The mathematics standards are presented in this document in chapter 3.

Florida’s reform effort is based on a commitment to continuous quality improvement in every school across the state. As such, it calls for improvement teams in schools to articulate a fundamentally new direction for instruction and to reexamine the ways in which the day-to-day business of schools is conducted.

A number of assumptions provide a foundation for Florida’s school improvement and accountability initiative. These include the following:

- All children can learn at high levels, given proper instruction in a supportive environment.
- All schools can be successful.
- The state focuses on accountability for student achievement; schools focus on schooling and instructional processes necessary to raise student achievement.
- Children’s health, safety, social, and educational needs must be met collaboratively by schools, parents, agencies, and the community.
- The education stakeholders closest to the learners are best able to determine the appropriate strategies to identify and solve school problems and to improve instruction.
- The individual school is the unit of educational accountability for improving student performance, and school-level public reporting of effectiveness is a critical component of accountability.
- Continuous quality improvement is “the way of work”: It results in a focus on education stakeholders, collegiality, teamwork, collaboration, responsiveness, flexibility, innovations, risk taking, and effectiveness.
The focus of Florida’s reform initiative is on what students need to know and be able to do for the 21st century.

The ultimate goal of education reform is to move from schooling that was designed in, and quite appropriate for, an industrial age to one that reflects and meets the needs of the new information age. Florida’s initiative invites schools to develop learning activities for students that deal with substantial, meaningful knowledge as it relates to performance in real life. Instead of teaching only content knowledge and skills, teachers must practice the difficult art of finding ways for each student to learn and to demonstrate that learning.

This current Florida education initiative differs from earlier approaches to school reform which were often characterized by detailed legislative mandates and minimum standards. This initiative represents a decentralized approach to reform. The state will hold schools accountable for high levels of student achievement. Local districts and schools are free to design learning environments and experiences that best help their unique students meet the Sunshine State Standards.

Education reform, then, is about developing the capacity at the local level to identify and solve problems related to raising student achievement. Raising student achievement requires both (1) raising expectations through high academic standards grounded in a foundation of reading, writing, and mathematics, applied in real-world contexts, and (2) improving the environment for effective teaching and learning based on the current research about how people learn.

**The Value of a Framework**

This curriculum framework is a resource and a guide for local education communities as they restructure their schools and improve their mathematics programs. Local planners who recognize the diversity of their students’ unique
learning styles, backgrounds, attitudes, interests, aptitudes, and needs know best what specific programs will help their students reach the Sunshine State Standards.

Grounded in national and state reform initiatives, this framework does not prescribe the specifics of classroom instruction. It presents broad, overarching concepts and ideas for development of curriculum and instruction. Curriculum guides will need to be developed at the local level to provide specific content and specific teaching, learning, and classroom assessment activities. They will need to be far more detailed than this framework and reflect the qualities and flavor of the community as well as the unique needs of the students in the community. This framework also provides overviews of instructional strategies and assessment that can help local educators create supportive, effective educational environments in which all students can achieve Florida’s high academic standards and benchmarks.

A statewide external assessment program will monitor student learning in reading, writing, mathematics, and thinking skills. This system will be based on the mathematics standards articulated in this curriculum framework and on the standards articulated in the language arts framework. However, in all subject areas, instruction must support the development of these essential skills.

To help local mathematics educators meet these challenges, this framework

- delineates for stakeholders what knowledge and skills the state will hold schools accountable for students learning at four developmental levels (grades preK-2, 3-5, 6-8, and 9-12);
- gives sample performance descriptions of how students might demonstrate these skills and knowledge, often in authentic, real-world contexts;
- correlates the sample performance descriptions to Florida’s Education Goal 3 Standards;
- encourages districts and schools to develop curricula guided by a locally developed vision designed to improve instruction through sound strategies and community support;
- promotes the selection and use of sound, well-developed, flexible, and innovative instructional strategies;
- provides overviews of models of good teaching, learning, and assessment that local education planners are encouraged to investigate and consider;
• presents ideas for developing connections within mathematics and with other disciplines;
• discusses the practical aspects of designing a quality learning environment;
• provides suggestions for the professional development of teachers; and
• includes suggestions and criteria for continuous district and school mathematics program improvement.

Florida’s school improvement and accountability initiative envisions more effective education for students in Florida’s public schools. This system describes a vision of learning and schooling that is innovative, yet sound; ambitious, yet feasible; rigorous for students and demanding of teachers, yet achievable. The ultimate goal is success for every student.

The Standards Movement

The current effort to develop national standards in various subject areas can be traced back to September 1989 when the nation’s governors recommended that America establish national education goals. Leading education reformers established goals through America 2000, later renamed Goals 2000, along with a plan to meet these goals. The National Council on Education Standards and Testing recommended the development of voluntary national standards. The National Council of Teachers of Mathematics led the way in the development of national standards; subsequently, standards have been developed in many other academic areas.

The Secretary’s Commission on Achieving Necessary Skills (SCANS) Report, developed by the U.S. Department of Labor, verified the need for a plan for educational reform. The Commission was charged with examining the demands of the workplace and determining whether the young people of the United States are prepared to meet those demands. Specifically, the Commission was directed to define the skills and competencies needed for employment, propose acceptable levels of proficiency, suggest effective ways to assess proficiency, and develop a strategy for assuring that the identified skills and competencies become a part of the learning opportunity for every American student.
The SCANS Report, *What Work Requires of Schools*, published in June 1991, defined the workplace competencies and foundational skills required for effective job performance in today's marketplace as well as for the future. This report has had a continuing impact on schools as they work to equip students with marketable skills. Florida's Schoolyear 2000 Initiative conducted research that verified the importance of these skills for Florida's job market. The SCANS competencies provide the basis for Florida's Education Goal 3 Standards.

**Standards Initiatives in Mathematics**


**How Was This Framework Developed?**

In response to the educational reform initiative reflected in *Florida's System of School Improvement and Accountability*, the Florida Department of Education began the development of a new design for curriculum frameworks in the fall of 1993. This new design is based on approaches being used in other states and was specifically based on a prototype document for science developed through the support of a National Eisenhower Curriculum Framework grant from the United States Department of Education.

In January 1994, a statewide advisory committee was formed, in cooperation with the Florida Organization of Instructional Leaders, to guide the framework activities. *The Principles Guiding the Development of Florida's New Curriculum Frameworks* was produced by this committee. The writing of draft frameworks in the areas of
language arts, mathematics, social studies, the arts, foreign languages, and health education/physical education, along with the revision of the science framework, was coordinated by the Department of Education through representative statewide writing teams for each subject area, under the leadership of curriculum specialists from the Department of Education. The writing teams conducted extensive research on content standards and instructional practices, received input from their professional organizations, deliberated issues, reached consensus, and crafted strong initial drafts.

The planning for this framework began with the selection of a committee representing fifteen of Florida’s school districts and all grade levels. Eleven mathematics teachers, six district mathematics supervisors, four post-secondary Mathematics Education faculty members, a Mathematics Education doctoral student and a Department of Education facilitator met over a period of six months to identify strands of mathematics and areas of focus to be covered in this document. Meetings were held to review other frameworks and to define the vision for mathematics in the state of Florida. Each PreK-12 strand was written by a team that included an elementary teacher, a middle school teacher, a high school teacher, and either a district supervisor or a college of education faculty member. The review process was initiated by presentations at five regional meetings held for all frameworks and presentations to the mathematics community at the Florida Council of Teachers of Mathematics (FCTM) Fall Leadership Conference and State Conference. In addition, draft copies were mailed to mathematics contacts in each district.

In 1995, systematic analysis of the drafts of the curriculum frameworks in mathematics and other subject areas was conducted to determine the extent to which each draft addressed the 
Principles, Florida’s System of School Improvement and Accountability
other major state initiatives, and national curriculum standards. The analysis also examined consistency in content, style, and format across the documents. The Center for Educational Technology (CET) at Florida State University and the Mid-continent Regional Educational Laboratory (McREL) Institute in Aurora, Colorado, conducted this analysis and developed a plan for revising and preparing the final versions of the documents. The McREL Institute was selected for this work because of its expertise in the analysis of standards for curriculum and because of its knowledge of national standards. With continued input from the original curriculum framework writing teams and experts, and the
assistance of CET and the McREL Institute, the revisions for each framework were prepared and reviewed.

Statewide reviews of the drafts were conducted through meetings of the original writing teams, focus groups of education stakeholders including business leaders and members of the Florida PTA, conference presentations, and mailings to each school district. The revisions were completed early in 1996. The new curriculum frameworks will provide assistance to all education stakeholders in their collaborative efforts to raise student achievement of Florida academic and work-related standards to world-class levels.

**KEY CHAPTER POINTS**

- Education reform is needed to keep pace with a changing world.
- Florida has created an educational reform initiative to raise student achievement to high levels.
- This initiative empowers schools to identify and solve problems at the local level.
- The Florida Curriculum Framework for Mathematics articulates state-mandated academic standards that raise expectations for student achievement. It also includes overviews of best practices in instruction for local educators to further investigate.
- This framework has drawn on standards initiatives at national and state levels.
Chapter 1: Visioning

CHAPTER HIGHLIGHTS
• The Importance of a Local Vision
• Creating a Vision: The Local Process
• Underlying Assumptions of a Vision for Mathematics Learning
• Mathematics Vision Statement

The Importance of a Local Vision

A vision is a vivid picture of the desired future: a detailed description of what should be, could be, and might become. Effective leaders and organizations need a clear vision of their goals if they wish to make real improvement. Similarly, Florida's education improvement initiative can best be realized if local community members come together to articulate a shared vision for educational excellence in their community.

Visioning is not about simply talking or writing about missions or goals; visioning uses words to create a dynamic picture of a new condition that will be intellectually and emotionally satisfying when achieved. Unless the stakeholders—educators, support staff, students, parents, and community members—understand the reasons for change and envision the desired changes in place, educational reform cannot happen. Once the picture of a new way of doing things in schools and classrooms is clearly in the minds of education stakeholders, they are often not content with the old ways.

Education leaders need to work with the community to create and communicate visions of improved schools, mathematics classrooms, and student achievement that education stakeholders can accept and work toward. In fact, if the vision is powerful, education stakeholders will think up new strategies along the way, find unexpected
resources, work beyond expectations, and make extraordinary things happen in order to fulfill their vision.

Creating a Vision: The Local Process

Real reform of education cannot take place unless local stakeholders share a vision of the future. Schools often develop a vision for their improvement efforts, but the visioning process does not have to stop there. Mathematics educators in every Florida school and district are also encouraged to develop and embrace a vision that defines their discipline, provides purpose and direction for improvement efforts, unifies the school community, and articulates the goals and value of a mathematics education.

All those interested in school improvement should contribute to the development of a school’s vision. Parents and guardians, business and community leaders, and other interested stakeholders are invited to join with students, educators, and other professionals in formulating a vision for substantial change. The intellectual and cultural diversity of the vision crafters will help ensure a strong, unique community vision for mathematics education. Involvement of all stakeholders in education builds ownership of both the process and the outcomes.

Vision crafters should focus their primary attention on how best to help their students reach Florida’s high academic standards. National, state, and local trends as well as best practices in curriculum, instruction, and assessment need to be considered. The vision described in this framework may also be helpful in the development of a vision for mathematics education in each local Florida school.

Underlying Assumptions of a Vision for Mathematics Learning

Certain underlying assumptions support the vision for mathematics education articulated in this framework. These include

- Every person is a learner; education professionals, students, and family form a community of learners.
- Effective teaching and learning connect concepts and processes to everyday events.
- A learning environment conducive to quality teaching and learning is the responsibility of the school community.
Learning takes place both in schools and in communities.
Cultural diversity enriches the learning environment.
Instructional programs and teaching strategies should accommodate
diverse learning styles and needs.
Excellence in mathematics teaching and learning grows from a
commitment shared by teachers, students, parents, administrators, and
the community at large.
Learning is a lifelong process. Successful learners are lifelong learners.

Mathematics Vision Statement

This vision for mathematics education is presented as a starting point, to encourage
local communities to develop mathematics visions for their students, their
classrooms, their schools, and their district.

In mathematics teaching and learning
• students are excited by, are interested in, and value their mathematical
  activities;
• students work together and individually to find solutions to real
  problems;
• technology and other tools are used as an integral part of the teaching
  and learning process;
• learning is conceptually based, meaningful, and connected within
  mathematics and with other disciplines, using real-world
  phenomena;
• the community has high but achievable expectations for
  all students;
• assessment is an integral part of the teaching and
  learning process;
• opportunities for both written and oral communication,
  as well as reflective thinking, are regularly integrated
  into the mathematics learning activities; and
• teaching strategies and learning environments
  promote mathematics equity for all students.
When members of a community work together to form a vision, they assess their programs and goals, discuss their options, and chart a course for action. A local vision of teaching and learning in mathematics reflects the highest ideals of a school community, serving to unify the community and to clarify its commitment to program improvement. Developing a local vision for improving mathematics education is an ongoing process, one that reflects the best of teaching, learning, and community values.

**Key Chapter Points**

- A vision is a picture created to describe the desired future.
- Visions unify a group by sensitizing everyone to the nature of commitment.
- Because they are products of communication, visions are neither static nor restrictive.
- The vision statement serves to inspire participants to believe that learning in mathematics can be different and better.
- Local educators are challenged to become actively involved in assuring the quality of mathematics education for all students.
- A vision statement helps generate a sense of deliberate and conscious effort in all that is done, serving to focus a community’s imagination and energy.
- The vision for mathematics developed by the statewide curriculum framework writing team can serve as a starting point for local communities to develop their own vision.
There are a number of general processes and abilities that are used in all subject areas. For example, locating information, organizing that information, and then using it to solve a problem or produce a product are useful abilities in virtually any area of study. Similarly, identifying the resources necessary for accomplishing a goal, setting milestones, and then managing those resources are abilities that are common to many subject areas. They are also important to success in everyday life at home, in the community, and in the workplace.

These practical but highly important cross-disciplinary processes and abilities have been identified as standards under Goal 3 in the document Florida’s System of School Improvement and Accountability. One of the seven goals that are the foundation for school reform in Florida, Goal 3 deals with student performance. It states,

Students successfully compete at the highest levels nationally and internationally and are prepared to make well-reasoned, thoughtful, and healthy lifelong decisions.

In all, eleven standards are identified within Goal 3, ten dealing specifically with student achievement. This chapter describes ways in which these ten general standards can be addressed in mathematics.

It is important to realize that the term standard is used somewhat differently in this chapter than it is in chapter 3. A Goal 3 standard describes a general category of processes and abilities that are important to all subject areas and the world of work.
The Sunshine State Standards described in chapter 3 of this framework refer to the knowledge and skills specific to mathematics.

Both the first ten standards of Goal 3 and the mathematics standards have been adopted by the State Board of Education and represent what the state will hold schools accountable for reaching. The Goal 3 standards can be summarized as follows:

**GOAL 3 STANDARDS**

1. Information Managers
2. Effective Communicators
3. Numeric Problem Solvers
4. Creative and Critical Thinkers
5. Responsible Workers
6. Resource Managers
7. Systems Managers
8. Cooperative Workers
9. Effective Leaders
10. Multiculturally Sensitive Citizens
11. Involvement of Families

In each subject area in the state of Florida, students will be expected to develop their skills and abilities as information managers, effective communicators, and so on. Indeed, Florida’s public schools are accountable to their stakeholders for students learning to apply the first ten standards of Goal 3 to all subject areas. Schools are expected to conduct assessments that will, along with external assessments conducted by the state on the first four standards, show that students are making progress toward Goal 3.

**Impact of Goal 3 Standards**

Many stakeholders will be affected by the teaching and assessment of Goal 3 standards. Students have a vested interest in understanding and attaining the Goal 3 standards, because these standards will affect their ability to function effectively in their personal and professional lives. Parents or other caregivers must participate in their children’s learning process and in the assessment of their children’s performance on Goal 3 standards. Standard 11 of Goal 3 calls on families to “share the
responsibility of accomplishing the standards set in Goal 3 throughout a student’s education from preschool through 12th grade.” School administrators and staff should welcome parents as full partners in helping students improve their academic performance by making time and opportunities for mutual communication available. Parents need to communicate with school personnel regarding curriculum, assessment, and goals for individual students, provide a home environment that is supportive of improving student performance, and provide encouragement and discipline as appropriate to support school success.

Teachers must assume new and different roles in assessment. New approaches to understanding student learning and performance will place teachers in the position of assessing student progress in more authentic ways. These expanded assessments should reflect how students will need to use content knowledge, as well as the Goal 3 general processes and abilities, in real life—now and in their future.

Florida’s school administrators have primary responsibility for encouraging, facilitating, and initiating changes within their schools. School administrators will be primarily responsible for identifying strategies for accessing teacher training offered by their district, the state, and other sources such as universities and colleges. Administrators’ primary responsibilities within the framework of Goal 3 assessment will be to support the integration of assessment and instruction in the classroom and establish school reporting systems for the multiple data sources that will be derived from Goal 3 assessment activities.

The business community stands to benefit greatly from the emphasis on Goal 3 standards. Indeed, the Goal 3 standards directly address skills effective workers need to be successful in the 21st century. The skills identified in the U.S. Department of Labor’s SCANS Report on necessary skills for the workplace are the basis of the Goal 3 standards. Consequently, Florida’s emphasis on the Goal 3 standards is an investment in the success of the business community.

**Using the Goal 3 Standards**

The Goal 3 standards do not exist in isolation; they should be an integral part of daily classroom instruction and assessment. To a great extent, the Goal 3 standards can be thought of as generic processes and abilities that help students apply specific mathematics content knowledge to real-world situations. As students learn
mathematics content, they are using the processes and abilities involved in being an information manager, effective communicator, numeric problem solver, and so on.

Teachers should directly address the processes and abilities involved in the Goal 3 standards. In fact, the Goal 3 processes and abilities can and should become a common “language” that is used in every classroom at every grade level. In this section, examples are provided to illustrate how each of the first ten standards can be used in mathematics. All of the examples depict activities that the teacher designs to help students learn and apply new knowledge to classroom and real-world activities. The designing of classroom tasks is one of the most important parts of the art of teaching. In the past, classroom activities often provided little flexibility in terms of the knowledge involved, what students do with that knowledge, and how students demonstrate their competence. The tasks designed around the Goal 3 standards should not be limiting. Each of the Goal 3 standards can play a significant role in tasks designed to integrate real-world problems and situations into classroom activities.

**Standard 1:** Florida students locate, comprehend, interpret, evaluate, maintain, and apply information, concepts, and ideas found in literature, the arts, symbols, recordings, video and other graphic displays, and computer files in order to perform tasks and/or for enjoyment.

Proficient information managers acquire, use, and manage information purposefully. Developing information managers involves creating tasks that require skills in information acquisition, use, and management. These tasks range from daily functions in school and work settings to everyday activities at home and in the community.

The infusion of technology and multimedia in various spheres of life has placed increased demands on information management skills. People frequently face challenges in locating, interpreting, applying, evaluating, and storing information. Numerous daily tasks require competence in Standard 1. Common examples include

- interpreting weather maps on television or in the newspaper;
- reading or giving directions to get to places;
- accessing information from data storage systems, such as electronic encyclopedias or atlases;
- setting up and operating a new appliance, such as a VCR;
following instructions to complete income tax returns;
keeping important documents and records organized;
interacting on electronic networks, such as the Internet; and
interpreting statistical data.

**Standard 2:** Florida students communicate in English and other languages using information, concepts, prose, symbols, reports, audio and video recordings, speeches, graphic displays, and computer-based programs.

**Effective communicators** convey thoughts, ideas, and information purposefully. Developing effective communicators involves creating tasks that require skills for transmitting and receiving communications. Communications are transmitted when a student speaks, writes, performs, or creates products. Communications are received by students through observing, reading, and listening—the skills of Standard 1. Media technology can significantly enhance communications.

To be competitive in the 21st-century global economy, students should be able to communicate effectively, not only in English, but also in one or more foreign languages. It is also important for students to be able to use languages pertinent to specialized areas, for example, mathematical notation and vocabulary, scientific language, Latin terminology, musical notation, and computer languages.

Communication is an essential form of human engagement. Success in the skills and abilities that are part of Standard 2 is vital to success in school, at home, and the workplace. Common examples of activities that involve communication skills include

- making a multimedia presentation to introduce a new marketing strategy;
- writing letters of application for jobs or educational programs;
- making formal or informal announcements;
- writing a technical report or a business plan;
- initiating and making conversation;
- writing or reciting a poem;
- viewing and listening to an opera or play; and
- discussing, as a member of a team or committee, ways to solve a problem.
Standard 3: Florida students use numeric operations and concepts to describe, analyze, disaggregate, communicate, and synthesize numeric data, and to identify and solve problems.

Numeric problem solvers analyze and solve mathematical or quantitative problems in applied situations in school, life, and the workplace. Developing numeric problem solvers involves creating tasks that require students to gather, read, manipulate, interpret, organize, and analyze quantitative data. Numeric problem solvers also verify, explain, and justify solutions to quantitative or mathematical problems. Students must be able to take advantage of technology such as calculators and computers that support mathematical problem solving. Common examples of activities that require competence in the skills and abilities of Standard 3 include

- understanding bus, train, and plane schedules;
- determining the best value of things to buy;
- keeping accounts and budgets for different purposes;
- measuring ingredients for recipes and distances for travel; and
- gathering, summarizing, and analyzing data to determine needs in particular situations.

Standard 4: Florida students use creative thinking skills to generate new ideas, make the best decision, recognize and solve problems through reasoning, interpret symbolic data, and develop efficient techniques for lifelong learning.

Creative and critical thinkers gather new information to answer questions and make conclusions, connections, and inferences from existing information. Creative thinking involves divergent thinking, originality, and the ability to find novel or unique relationships and solutions. Creative thinkers have a high tolerance for ambiguity; they seek out opposing viewpoints.

Developing creative and critical thinkers involves creating tasks that require students to become proficient in using critical and creative thinking.
processes to solve problems. As they progress through their school years, students are expected to apply various problem-solving processes to the scientific method, logical analysis, trial-and-error techniques, and the creation of functional objects, works of arts, and performances. Students also must be able to creatively deal with limitations imposed upon the creative process, such as space limitations or lack of availability of materials. Teachers should nurture attitudes of persistence and perseverance during problem-solving activities.

**Standard 5:** Florida students display responsibility, self-esteem, sociability, self-management, integrity, and honesty.

In order to develop **responsible workers**, educators should emphasize the personal and social attributes that form positive social skills, such as self-management behaviors, self-esteem, and honesty. These attributes are used in day-to-day interactions with people in school, at home, in the community, and in the workplace.

Unlike Standards 1 to 4, which focus on cognitive and academic development, Standard 5 emphasizes affective and social growth as well as self-discipline. Instruction in the skills and abilities of Standard 5 occurs in formal and informal interactive settings. Teachers, parents, the school community, and the community at large should work as partners to develop students as responsible workers. The learning environment must be conducive to nurturing the personal and social attributes that define Standard 5. Positive behaviors can be reinforced through consistent role modeling by peers and adults. Mentoring, counseling, individual educational plans, and contracts between teachers and students are effective ways to help students become responsible workers.

**Standard 6:** Florida students will appropriately allocate time, money, materials, and other resources.

Developing effective **resource managers** involves helping students learn to allocate and manage resources to complete projects and tasks. Instruction in and assessment of the skills and abilities delineated in Standard 6 occurs as students prepare action plans to accomplish tasks, allocate time and necessary resources, implement plans, and evaluate whether or not the resources allocated were adequate. Students can demonstrate their effectiveness as resource managers at home, in school and school-
related activities, in the community, and in the workplace.
The intent of Standard 6 is to help students become proficient in allocating time, preparing and following time lines, preparing budgets, and acquiring and distributing materials and other resources, such as facilities, technology, or environmental resources. These skills can be used when conducting research, developing products, or preparing presentations.

**Standard 7:** Florida students integrate their knowledge and understanding of how social, organizational, informational, and technological systems work with their abilities to analyze trends, design and improve systems, and use and maintain appropriate technology.

Developing proficient systems managers involves helping students understand what systems are, how they work, and how to use the systems approach to solve problems or design solutions. Instruction in and assessment of the skills and abilities of Standard 7 occur as students analyze information and solve problems that help them see the big picture and its parts.

The intent of Standard 7 is to help students use the systems approach as a way of getting a better grasp of events and phenomena in their world. Thus, helping students learn about the natural systems of science, the systems of language, and systematic mathematical thinking is a good way to introduce the concept of systems. Efficient systems managers use systems concepts to process information, solve problems, develop new models, or change existing systems to produce better results.

Various concepts can be studied using the systems approach. Students should be able to identify and understand natural, social, organizational, informational, and technological systems. Systems in their world include grading systems, the education system, the lunchroom system, computer systems, government systems, and the judicial system.

**Standard 8:** Florida students work cooperatively to successfully complete a project or activity.

In order to develop cooperative workers, educators should emphasize the attributes and interpersonal skills necessary to work effectively in teams, a process that is used extensively in the work world. The goal is to develop students and workers who can interact cooperatively and productively in groups.
Unlike Standard 5 (responsible workers), which deals with affective and social growth on a personal level, Standard 8 deals with goal- or task-oriented social behaviors that involve group work. To help develop cooperative workers, opportunities must be provided for students to perform tasks in cooperative groups. Such opportunities should help students understand group processes, assume various roles in the group, keep the group on task, motivate the group toward task completion, and evaluate the effectiveness of the group in accomplishing goals. Instruction in the skills and abilities of Standard 8 might occur in classroom, community, or workplace-like settings.

**Standard 9:** Florida students establish credibility with their colleagues through competence and integrity, and help their peers achieve their goals by communicating their feelings and ideas to justify or successfully negotiate a position that advances goal attainment.

In order to develop effective leaders, educators should emphasize the attributes and interpersonal skills necessary for students to advance group and individual goals. Students must learn to develop skills in listening, communicating, decision making, conflict resolution, and negotiation. This standard aims to develop students who can lead groups productively.

Standard 9 is closely related to Standard 5 (responsible workers), which deals with affective and social growth on a personal level, and Standard 8 (cooperative workers), which deals with goal- or task-oriented group behaviors. In order to help develop effective leaders, opportunities must be provided for students to take on leadership responsibilities in safe, nonthreatening environments. Such opportunities should help students learn to communicate directly, treat individuals fairly, and separate work- and group-related issues from personal ones.

**Standard 10:** Florida students appreciate their own culture and the cultures of others, understand the concerns and perspectives of members of other ethnic and gender groups, reject the stereotyping of themselves and others, and seek out and utilize the views of persons from diverse ethnic, social, and educational backgrounds while completing individual and group projects.

In order to develop multiculturally sensitive citizens and workers, educators should help students become knowledgeable about their own cultural backgrounds.
and the cultures of others. Instruction in and assessment of the skills and abilities identified in Standard 10 should help students understand the importance of treating others with dignity and respect. This standard involves broadening students’ knowledge and understanding of the languages, customs, beliefs, traditions, and values of different cultures.

**Standard 11:** Families will share the responsibility of accomplishing the standards set in Goal 3 throughout a student’s education from preschool through 12th grade.

Educators are encouraged to invite and facilitate the *involvement of families* in their children’s education. Parents should be encouraged to volunteer in the classroom, help at home with homework and projects, monitor progress through parent-teacher conferences, generate community support for education, and model lifelong learning.

**Suggestions for Mathematics Educators**

Schools will be held accountable for incorporating the Goal 3 student-achievement standards into instruction and classroom assessment. The following are examples of mathematics classroom activities that integrate the Goal 3 standards:

In conjunction with a laboratory activity in science, students use exponential functions to study the growth pattern of living things. Working in groups, they gather data from their experiment, graph the results, and apply the concept of exponential growth as they attempt to create mathematical models for their findings. The groups make class presentations in which they discuss the exponential function which models the growth and practical applications for their research.

This example involves Standard 1, information managers; Standard 2, effective communicators; Standard 3, numeric problem solvers; Standard 4, creative and critical thinkers; and Standard 8, cooperative workers.

Small groups of students choose sides to debate “The world really does/does not need fractions.” Each group must prepare a convincing argument with supporting data for their position.

This example involves Standard 3, numeric problem solvers; Standard 4, creative and critical thinkers; and Standard 8, cooperative workers.
In conjunction with a social studies unit on the issues of the elderly, students devise financial plans for old age. Using figures on income levels provided by the instructor or found on the Internet, students design plans using their knowledge of interest rates, financial markets, and government savings plans. Students also consider factors such as potential catastrophic illnesses, individual health needs, and the particulars of a family situation. After formulating their plans, students pair up to discuss and compare their plans. During the discussions, students carefully consider their partners’ plans and offer constructive feedback. Students are encouraged to consider the sensitivity of issues associated with financial planning.

This example involves Standard 3, numeric problem solvers; Standard 4, creative and critical thinkers; Standard 5, responsible workers; Standard 7, systems managers; and Standard 10, multiculturally sensitive citizens.

Working in teams, students must find the carrying capacity of a local landfill. Each team elects a leader who will oversee the project. The students conduct the research necessary to determine the volume of the site, the available space, and how long before the landfill reaches capacity. Leaders create a work schedule with input and suggestions from team members, coordinate their teams’ research, and report progress to their instructor. Each team presents its findings to the class and guests representing the local waste and landfill management system.

This example involves Standard 1, information managers; Standard 2, effective communicators; Standard 3, numeric problem solvers; Standard 6, resource managers; Standard 7, systems managers; Standard 8, cooperative workers; and Standard 9, effective leaders.

**Key Chapter Points**

- The first ten standards of Florida’s Goal 3 Standards are general processes and abilities that cut across all subject areas.

- These processes and abilities are important to success in school and in everyday life at home, in the community, and in the work world.

- These Goal 3 Standards should be an integral part of daily classroom instruction and assessment in every subject area at every grade level; they will help students
apply specific content knowledge in real-world situations.
Chapter 3: Mathematics
Sunshine State Standards

CHAPTER HIGHLIGHTS
• A New Approach to the Mathematics Curriculum
• The Hierarchic Structure of Strands, Standards, and Benchmarks
• Introduction to the Mathematics Strands
• Mathematics Sunshine State Standards

The standards and benchmarks for mathematics represent the heart of the Florida Curriculum Framework for Mathematics because high standards are the center of the efforts to reform and enhance education in Florida. Before addressing the mathematics standards, it is useful to consider why we need academic standards. In her book *National Standards in American Education: A Citizen’s Guide*, Diane Ravitch, former Assistant Secretary of Education at the U.S. Department of Education, explains that standards are a necessary and accepted part of American life in almost every field but education:

> Americans clamor for standards in nearly every part of their lives. They expect strict standards to govern construction of buildings, bridges, highways, and tunnels; shoddy work would put lives at risk. They expect explicit standards in the field of telecommunications; imagine how difficult life would be if every city, state, and nation had incompatible telephone systems. They expect stringent standards to protect their drinking water, the food they eat, and the air they breathe…. Even the most ordinary transactions of daily life reflect the omnipresence of standards. (pp. 8-9)

Standards have the potential of affecting many aspects of schooling in Florida. The mathematics curriculum—what teachers teach and how they teach it—should be organized around the mathematics standards. Assessment is one of the most obvious areas that will be affected. The state will be assessing reading, writing, and mathematics based on the language arts and mathematics curriculum frameworks.
However, on the local level, the state standards for mathematics should form the basis of classroom assessments for mathematics. Finally, the systems used to report student progress—report cards and transcripts—should have a clear relationship with these academic standards. In short, the mathematics standards presented in this framework should be the starting point for mathematics education in Florida’s education system. This chapter presents those standards in detail.

The key issue in mathematics education is not whether to teach fundamentals, but which fundamentals to teach and how to teach them.


A New Approach to the Mathematics Curriculum

Changes in society and technology have greatly impacted what is and is not fundamental for a mathematically literate adult. To prepare our students for a changing world, the content of traditional curriculum, its organization, and presentation must be adapted to better meet the needs of students. Currently, content strands are arranged horizontally. For example, children first study arithmetic, then basic algebra, then geometry, followed by more algebra, and finally—for those who have successfully negotiated this layered approach—calculus. This layer-like mathematics curriculum creates increasingly dense filters through which students must pass before progressing on to other mathematics topics. These filters may impede rather than encourage children’s growth in mathematics learning.

In contrast to the layered approach described above, this curriculum framework provides parallel strands that allow for the development of significant mathematical ideas from informal intuitions to scientific or mathematical research of real-world problems. Beginning in the primary grades, students should have opportunities to continually revisit these strands as they progress through school.

The Hierarchic Structure of Strands, Standards, and Benchmarks

The standards presented in this chapter have a specific hierarchic structure. There are several levels of information, each more specific than the next.

Subject area = domain, content area, such as language arts, mathematics, science, music
The strands, standards, and benchmarks make up the Sunshine State Standards. These have been adopted by the State Board of Education as a rule, 6A-1.09401, FAC. This rule requires public schools to provide appropriate instruction to assist students in the achievement of these standards. Each district school board must incorporate the Sunshine State Standards into the district Pupil Progression Plan.

A strand is the most general type of information. A strand is a label for a category of knowledge under which standards are subsumed. For example, there are five strands in mathematics:

- **Strand A:** Number Sense, Concepts, and Operations
- **Strand B:** Measurement
- **Strand C:** Geometry and Spatial Sense
- **Strand D:** Algebraic Thinking
- **Strand E:** Data Analysis and Probability

Each of these strands contains two or more standards. A standard is a description of general expectations regarding knowledge and skill development within a strand. For example, within mathematics Strand A: Number Sense, Concepts, and Operations, there are five mathematics standards:
Standard 1: The student understands the different ways numbers are represented and used in the real world.

Standard 2: The student understands number systems.

Standard 3: The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

Standard 4: The student uses estimation in problem solving and computation.

Standard 5: The student understands and applies theories related to numbers.

These mathematics standards provide more specific guidance concerning what students should know and be able to do in relationship to the Number Sense, Concepts, and Operations strand.

The most specific level of information is the benchmark. A benchmark is a statement of expectations about student knowledge and skill at the end of one of four developmental levels: grades PreK-2, 3-5, 6-8, and 9-12. Benchmarks translate mathematics standards into expectations at different levels of student development. Within a standard, one would expect high school students to be performing differently from primary students. The benchmarks describe these differing levels of expectations. Although the identified developmental levels span several grades in order to accommodate continuous progress approaches, the benchmarks describe expected achievement as students exit the developmental level, that is, at the end of second grade, at the end of fifth grade, at the end of eighth grade, and at the end of twelfth grade. It is expected that several benchmarks might often be combined in a single teaching or assessment activity. The listing of separate benchmarks should not be construed to mean that students must demonstrate achievement of them one at a time, to be checked off by the teacher.

Expectations of student knowledge and skills are described in the benchmarks, but the benchmarks are also written with some assumptions regarding student learning. Although knowledge and skills stated at an earlier level of schooling might not be reiterated within benchmarks at later levels, they remain important and should be reinforced and even retaught, if necessary. For example, in the early years, if students
are expected to master the fundamentals of place value, learning and assessments in later grades should also incorporate this skill, even though the expectation is not explicitly restated within benchmarks for the later years. It is also assumed that in meeting the expectations described in these benchmarks, students are working with material that is developmentally appropriate with regard to their age, developmental level, and grade level.

Accompanying the benchmarks are sample performance descriptions. These sample performance descriptions suggest how teachers might ask students to apply the knowledge and skill described in the benchmark. For example, consider the following benchmark at the 3-5 level within mathematics Strand A, Standard 2:

The student recognizes and compares the decimal number system to the structure of other number systems such as the Roman numeral system or bases other than ten.

The sample performance description that accompanies this benchmark is

Achievement of the benchmarks may be demonstrated when the student compares the characteristics of other number systems (such as Roman, Mayan, Egyptian, base 2, and modular) and, in small groups, develops a new number system and explains how the basic operations work within that system.

To perform this activity, students must apply the knowledge and skill described in the benchmark.

Each sample performance description is keyed to specific Goal 3 standards; for example, in the above sample performance description, students are using the processes and abilities associated with Goal 3 Standards 1, 2, 3, 4, 7, 8, and 10. In addition, these sample performance descriptions incorporate Goal 3 performance at the appropriate developmental levels. In chapter 2, Goal 3 standards were described as an integral part of Florida education. The first ten standards within Goal 3 are to be integrated into each content area.

The sample performance descriptions and their Goal 3 correlations are meant to suggest to local curriculum and assessment developers and teachers the kinds of classroom assessment activities that can be used with the benchmarks. They are not one-to-one assessment items for the benchmarks; neither are they state-mandated
assessments. They serve only to suggest to local curriculum and assessment designers and teachers how they might begin to think about ways to determine if students are achieving or are making adequate progress toward achieving the benchmarks. They also provide examples of ways in which to integrate knowledge and skills from other content areas. As districts implement these frameworks, it is anticipated that more sample performance descriptions will be developed that are grade specific and cover the scope of the benchmarks. Designers and teachers should choose the content, topic, or processes for the activities appropriate to the local curriculum and develop completely new performance descriptions.

For ease of reference, the table of standards and benchmarks uses an identification system that mirrors the hierarchic structure just described. Each strand, standard, benchmark, and sample performance description has been assigned a unique identification code. The codes associated with the benchmarks and sample performance descriptions reflect the structure of this coding system. For example, note the following benchmark:

\[ \text{MA.E.1.4.2} \]

\[ \text{The student calculates measures of central tendency (mean, median, and mode) and dispersion (range, standard deviation, and variance) for complex sets of data and determines the most meaningful measure to describe the data.} \]

This code indicates that the benchmark is in the content area of mathematics (MA) under strand E, Data Analysis and Probability. The next two numbers identify the standard (1) under which the benchmark is categorized, and the developmental level (4) designated for this benchmark, that is, grades 9-12. The last number, 2, signifies that this is the second benchmark found under the standard at this developmental level. Sample performance descriptions share a similar identification code but differ in having a lowercase letter appended. This can be seen in the code for a sample performance description associated with the benchmark above:

\[ \text{MA.E.1.4.2.a} \]

\[ \text{Achievement of the benchmarks may be demonstrated when the student prepares a statement describing the effect of a 3.8\% raise to all employees on both the mean and standard deviation of both expenses and profit.} \]
The letter “a” indicates that this is the first sample performance description provided for this benchmark.

In addition to the coding system, the layout of the table of mathematics strands, standards, and benchmarks reflects the hierarchic structure: each new strand, standard, and benchmark level begins a new page. This offers an easy way for teachers to re-sort and organize the material by development level.

The standards and benchmarks in the curriculum frameworks identify the essential knowledge and skills that students should learn, for which the state will hold schools accountable. Nevertheless, how the standards and benchmarks are organized, what specific curriculum, instructional strategies, materials, and activities are designed to teach them, how much time is spent teaching them, and when they are taught within the developmental levels are local decisions.

Introduction to Strand A: Number Sense, Concepts, and Operations

An important goal of mathematics instruction is to develop students’ ability to reason intelligently with quantitative information. Therefore, mathematics curriculum should be designed to prepare students to use their knowledge of number in flexible and creative ways, not just in routine, predictable calculations. In this respect, the emergence of calculators and computers as powerful tools for representing and manipulating quantitative information has presented a formidable challenge to more traditional forms of instruction.

Number sense is often described as an intuition about numbers and their relationships. This includes having a feeling for comparisons among numbers, a knowledge of the effects of various operations on numbers, the ability to represent numbers in several ways, and the skills to interpret and use numbers from real-world situations.

School experiences likely to develop students’ ability to reason quantitatively should be developed through exploratory experiences which relate numbers to a students own world. When the development of number sense, operation sense, and estimation skills becomes an integral part of classroom experiences, children gain confidence and develop perceptive, inquiring minds necessary to solve problems that they will encounter as members of our ever-changing society—a society that will depend on its citizens being both literate and numerate.
Introduction to Strand B: Measurement

Activities involving measurement give children numerous opportunities to explore, organize and make sense of their world as well as provide for active participation in and exploration of topics in other disciplines. Lynn Stein, in his book *On the Shoulders of Giants* (1990), extends the connections of measurement to other topics within mathematics and elaborates on the importance of measurement within the curriculum. He writes,

Experiences with geometric quantities (length, area, volume), with arithmetic quantities (size, order, labels), with random variables (spinners, coin tosses, SAT scores), and with dynamic variables (discrete, continuous, chaotic) all pose special challenges to answer a very child-like question: “How big is it?” One sees from many examples that this question is fundamental...Students who grow up recognizing the complexity of measurement may be less likely to accept unquestionably many of the common misuses of numbers and statistics (p. 6).

Students are frequently exposed to measurement by rigidly applying formulas to well-contrived situations that hold little if any meaning for them. Such an emphasis on formulas cannot prepare children to deal intelligently with measurement that occurs in real situations where approximation and estimation are often necessary, where errors in measurement are likely to occur and must be accounted for, and where geometrical shapes are often irregular or unfamiliar. Students can develop their understanding of measurement and systems of measurement through experiences which enable them to use a variety of techniques, tools, and units of measurement. These situations should include both standard and metric systems so that students become fluent in both systems, without the contrived conversions from one system to another.
Introduction to Strand C: Geometry and Spatial Sense

Children’s awareness of geometry begins at an early age when they play with blocks and puzzles. The enthusiasm that is displayed at this age can be encouraged through the years with the use of rich, hands-on geometric experiences. These experiences will enable students to improve their spatial sense and other mathematical skills.

Geometry activities lend themselves to cooperative learning situations. This is a way to assist students in developing their communication skills and their self-confidence. The increase in self-confidence will enable students to be successful with the study of formal geometry and encourage them to continue with the study of higher levels of mathematics.

Technology must be introduced as a tool for the students’ use at an early age and must continue to be used on a regular basis. All students should be given opportunities to use manipulatives and computer graphics software as well as opportunities to participate in problem solving activities through group and individual investigations. This will enable students to recognize, visualize, represent, and transform geometric shapes and to use their knowledge of geometric properties, relationships, and models in other areas of mathematics and in describing and analyzing the physical world.

Connections between geometry and other branches of mathematics should be shown continually throughout the education process. Connections to other disciplines should also be part of making geometry come alive for the students. If technology and connections become the common threads in geometry instruction, then students will appreciate and respect mathematics in our world today and in the future.

Introduction to Strand D: Algebraic Thinking

From the simple rhythm of a nursery rhyme, to the complexity of a symphony, from the toss of a child’s ball, to the orbit of a spacecraft, mathematics can be used to find patterns and express relationships. From their first educational experiences, students are encouraged to look for patterns in the world around them.

Students can develop their understandings of the concepts associated with functions, patterns, and relationships through experiences which enable them to describe,
discover, represent, and analyze relationships among variable quantities and to apply them to solve meaningful problems.

Students should be encouraged to observe and describe many kinds of patterns in the world around them. Students can draw upon these experiences to explore properties of algebraic relations. They should investigate patterns leading to formulas from home and work contexts, from other disciplines, as well as from other strands, and understand them as examples of mathematical relationships. The exploration of functional relationships leads to understandings of cause and effect relationships essential to solving many real-world problems.

Through the use of manipulatives and technology, students can model problems and find solutions based on observed patterns and relationships, expressing the process symbolically and verbally. Technology and manipulatives also facilitate the transition from concrete modeling to abstract reasoning, thus increasing the depth and breadth of students’ mathematical knowledge. As students develop confidence in representing and solving problems, they should extend these skills to more abstract and symbolic representations.

**Introduction to Strand E: Data Analysis and Probability**

The current information age is full of quantitative information. This information affects decisions related to health, citizenship, employment, personal and professional finance, sports, and many other activities. To be an informed and productive citizen today, a person must have the ability to make intelligent decisions related to available data. As modern society becomes even more technical, the processes of collecting, organizing, describing, and interpreting data, as well as making decisions and predictions on the basis of that information will become increasingly important.

Probability and statistics permeate almost all disciplines, and their study allows us to make sense of our experiences in a wide variety of ways. Because of this, probability and statistics is an avenue through which children can explore the world around them. The recent advances in technology have also put within children’s grasp concepts and mathematical knowledge available to few just years ago. Technology allows children to locate, organize, summarize, manipulate, and display
large quantities of information which they can use for prediction and interpretation and further study.

Students learn through experiences that enable them to systematically collect, organize, and describe sets of data. Students should also have the opportunity to use probability to model situations involving random events and to make inferences and arguments based on analysis of data and mathematical probabilities.
Summary of Strands and Standards for Mathematics

A. Number Sense, Concepts, and Operations
   1. The student understands the different ways numbers are represented and used in the real world.
   2. The student understands number systems.
   3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.
   4. The student uses estimation in problem solving and computation.
   5. The student understands and applies theories related to numbers.

B. Measurement
   1. The student measures quantities in the real world and uses the measures to solve problems.
   2. The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).
   3. The student estimates measurements in real-world problem situations.
   4. The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

C. Geometry and Spatial Sense
   1. The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.
   2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.
   3. The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.

D. Algebraic Thinking
   1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.
   2. The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.
A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

<table>
<thead>
<tr>
<th>Level</th>
<th>Benchmark</th>
<th>Sample Performance Descriptions</th>
<th>Goal 3 Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grades PreK-2</td>
<td>MA.A.1.1.1 associates verbal names, written word names, and standard numerals with the whole numbers less than 1000.</td>
<td>MA.A.1.1.1.a uses a hundreds chart and counters (paper squares, cubes, or bingo chips) to investigate counting patterns and patterns in number arrangements, while practicing saying and writing numbers.</td>
<td>1, 2, 4</td>
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<tr>
<th>Hundreds Chart</th>
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<td>1 2 3 4 5 6 7 8 9 10</td>
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<td>11 12 13 14 15 16 17 18 19 20</td>
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A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

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<td>Grades PreK-2</td>
<td>MA.A.1.1.2 understands the relative size of whole numbers between 0 and 1000.</td>
<td>MA.A.1.1.2.a collects leaves for a science project and estimates how many have been collected. The student then counts to see how accurate the estimate is, sorts the leaves by size, and arranges the sorted piles in order from largest pile to smallest pile. The student orders actual counts numerically from least to greatest. MA.A.1.1.2.b uses cards with numbers 3, 6, and 8 to make the greatest 2-digit number possible, then the least 2-digit number possible.</td>
<td>1, 4</td>
</tr>
<tr>
<td></td>
<td>MA.A.1.1.3 uses objects to represent whole numbers or commonly used fractions and relates these numbers to real-world situations.</td>
<td>MA.A.1.1.3.a solves the following problem using symbols to represent quantities and answers related questions: ● = one strawberry. Parker needs ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ; she has ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●. How many more does Parker need? Then the student draws the correct number of strawberries.</td>
<td>2, 3, 4</td>
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A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

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| Grades PreK-2 | MA.A.1.1.3 (continued) uses objects to represent whole numbers or commonly used fractions and relates these numbers to real-world situations. | MA.A.1.1.3.b determines the appropriate prizes in a school contest and draws each prize. Given the following prizes: \[
\begin{align*}
\frac{1}{4} \text{ pizza} & \quad \frac{1}{3} \text{ pizza} & \quad \frac{1}{5} \text{ pizza} & \quad \frac{1}{2} \text{ pizza}
\end{align*}
\] the student answers these questions: Which should be the first-place prize? Why? Which should be the second-place prize? Why? Which should be the last-place prize? Why? Which should be the third-place prize? Why?
| 1, 2, 3, 4 |

**Solution:**

\[
\begin{align*}
\frac{1}{2} & \quad \text{first place, because it’s the largest} \\
\frac{1}{3} & \quad \text{second place, because it’s the second largest} \\
\frac{1}{4} & \quad \text{third place, because it’s the next to the smallest} \\
\frac{1}{5} & \quad \text{last place, because it’s the smallest}
\end{align*}
\]
A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

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<tr>
<td>Grades PreK-2</td>
<td>MA.A.1.1.4 understands that whole numbers can be represented in a variety of equivalent forms.</td>
<td>MA.A.1.1.4.a represents the number 5 in four different ways.</td>
<td>3, 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Sample Solutions:</strong> 5, 4 + 1, 3 + 2, 7 - 2</td>
<td></td>
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### A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

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<td>Grades 3-5</td>
<td>MA.A.1.2.1 names whole numbers combining 3-digit numeration (hundreds, tens, ones) and the use of number periods, such as ones, thousands, and millions, and associates verbal names, written word names, and standard numerals with whole numbers, commonly used fractions, decimals, and percents.</td>
<td>MA.A.1.2.1.a inserts digits from 345,678,129 into a number period chart and reads the number.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MA.A.1.2.2 understands the relative size of whole numbers, commonly used fractions, decimals, and percents.</td>
<td>MA.A.1.2.2.a makes a model to show how numbers are related in a contextual situation. Example: The student finds out the capacity of Joe Robbie Stadium in Miami (home of the Dolphins), Tampa Stadium in Tampa (home of the Buccaneers), and the Gator Bowl in Jacksonville (home of the Jaguars), and compares the capacity of the stadium to the population of the city. The student then expresses the ratio as a fraction and a decimal and answers the following question: In which city could the largest percentage of the population attend a football game? The student then explains his or her answer.</td>
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A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

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<td>Grades 3-5</td>
<td>MA.A.1.2.3 understands concrete and symbolic representations of whole numbers, fractions, decimals, and percents in real-world situations.</td>
<td>MA.A.1.2.3.a uses pennies to model percents. The student counts out 100 pennies, all heads up, and turns over enough pennies to show that 26% of the hundred pennies are heads down. The student answers the question: How many more pennies need to be turned over to show 94% heads down?</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA.A.1.2.3.b answers questions such as: What fraction of the class has brown hair? What fraction of the class has a cat? What fraction of the class is taller than 4 feet?</td>
<td>2, 3</td>
</tr>
</tbody>
</table>
## A. Number Sense, Concepts, and Operations

1. The student understands the different ways numbers are represented and used in the real world.

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</table>
| Grades 3-5 | MA.A.1.2.4 understands that numbers can be represented in a variety of equivalent forms using whole numbers, decimals, fractions, and percents. | MA.A.1.2.4.a demonstrates, through drawings or concrete items, that 2 quarters can be represented as .50 or 1/2 of a dollar.  
MA.A.1.2.4.b writes a short paper explaining how fractions and decimals are related.  
MA.A.1.2.4.c states whether the given form of a number is the best one to use for a given situation. The student converts the number to a better form and explains why the new form is better. Fractional quantities can be expressed as fractions, mixed numbers, and decimals.  
**Example:**  
Change from a purchase: \(2 \frac{3}{10}\) dollars  
Loaves of bread required to make picnic sandwiches: \(1 \frac{1}{2}\) loaves  
Amount of pizza left over from lunch: 0.25 pizza  
**Sample Solutions:**  
Change from a purchase should be $2.30. Bread is represented appropriately. Pizza should be a fraction--1/4 of the pizza left. | 1, 2, 3, 4  
1, 2, 4  
1, 2, 4 |
A. **Number Sense, Concepts, and Operations**

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<tr>
<td>Grades 6-8</td>
<td>MA.A.1.3.1 associates verbal names, written word names, and standard numerals with integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; radicals; absolute value; and ratios.</td>
<td>MA.A.1.3.1.a draws a picture and describes how an elevator could be used to explain integers to a friend.</td>
<td>2, 3, 4</td>
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A. Number Sense, Concepts, and Operations

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<tr>
<td>Grades 6-8</td>
<td>MA.A.1.3.2 understands the relative size of integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; radicals; absolute value; and ratios.</td>
<td>MA.A.1.3.2.a uses data displays to demonstrate an understanding of the relative size of fractions and percents and answers questions about a given problem. <strong>Example:</strong> In the pictograph below, the striped sector represents the portion of time that an average teenager of Andrew Lewis Middle School spends on the phone on the weekends. About what fraction of the weekend do these teenagers spend on the phone? About what percent of the weekend do they spend talking on the phone?</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA.A.1.3.2.b determines the greatest distance from the sun that each planet in the solar system reaches. Then the student expresses the ratio of each distance to that of the earth, using scientific notation.</td>
<td></td>
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### A. Number Sense, Concepts, and Operations

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<td>Grades 6-8</td>
<td>MA.A.1.3.3 understands concrete and symbolic representations of rational numbers and irrational numbers in real-world situations.</td>
<td>MA.A.1.3.3.a explains the need for absolute value in a contextual situation. Example: Below is the graph of the path of a pool ball when hit against the side of the table. Which equation accurately represents the line? Explain your choice.</td>
<td>1, 2, 3, 4, 7, 8</td>
</tr>
<tr>
<td></td>
<td>MA.A.1.3.4 understands that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, radicals, and absolute value.</td>
<td>MA.A.1.3.4.a represents numbers in a variety of ways and answers questions about a given problem. Example: Batting averages for baseball and softball players are reported as a three-digit decimal that is found by dividing the number of hits by the number of times at bat. If Beau has a batting average of .280 and has been at bat 25 times, how many hits does he have? What will his average be if he gets a hit on his next time at bat? What would a batting average of 1.000 mean? How many consecutive hits would he need to have a batting average of .500?</td>
<td>2, 3, 4</td>
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A. Number Sense, Concepts, and Operations

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<tr>
<td>Grades 9-12</td>
<td>MA.A.1.4.1 associates verbal names, written word names, and standard numerals with integers, rational numbers, irrational numbers, real numbers, and complex numbers.</td>
<td>MA.A.1.4.1.a finds the size of the national debt and the latest U.S. population and uses the information to describe what the figures mean to the individual citizen.</td>
<td>1, 2, 3, 4</td>
</tr>
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<td></td>
<td>MA.A.1.4.2 understands the relative size of integers, rational numbers, irrational numbers, and real numbers.</td>
<td>MA.A.1.4.2.a solves expressions provided on index cards and, as a member of a group, makes a Human Number Line at the front of the room, with each student assuming her or his appropriate place on the line.</td>
<td>1, 2, 3, 4, 8</td>
</tr>
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Example:
The cards have number representations or algebraic expressions such as:

a) If \( x^2 = 130 \), find \( x \)  
b) \( \sqrt{-50} \)  
c) \( |-5| \)  
d) slope of a horizontal line  
e) \(-2 \frac{1}{2}\)  
f) \( \frac{3}{\sqrt{27}} \)

The student computes the expression given on the card and then stands in the appropriate place on the number line, to reinforce that the answers are numbers. The student then discusses how the answers relate to the original numbers on the cards.
A. Number Sense, Concepts, and Operations

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| Grades 9-12 | MA.A.1.4.3 understands concrete and symbolic representations of real and complex numbers in real-world situations. | MA.A.1.4.3.a determines whether calculated numbers are rational or irrational numbers. Example: Given the formula for braking distance—

\[ s = \sqrt{30d} \]

where \( s \) is the speed of the car when the brakes are applied and \( d \) is the length of the skid marks, the student answers these questions: For what \( d \) will \( s \) be a rational number? For what \( d \) will \( s \) be an irrational number? Is there a \( d \) for which \( s \) will be a complex number? The student explains his or her answers. | 2, 3, 4 |
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<tr>
<td>Grades 9-12</td>
<td>MA.A.1.4.4 understands that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, radicals, absolute value, and logarithms.</td>
<td>MA.A.1.4.4.a writes the radius of the earth (about 6.37 million meters) in two other ways. MA.A.1.4.4.b explains the relationship of the rating system to the relative intensity of earthquakes or hurricanes and answers questions about a given problem. Given this scenario: an earthquake measuring 3 on the Richter scale hit the east coast of the United States; the following year, an earthquake measuring 7 on the Richter scale hit Asia. How many times more intense was the earthquake in Asia than the one in the United States?</td>
<td>3, 1, 2, 3, 4</td>
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## A. Number Sense, Concepts, and Operations

2. The student understands number systems.

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<td>Grades PreK-2</td>
<td>MA.A.2.1.1 understands and applies the concepts of counting (by 2s, 3s, 5s, 10s, 25s, 50s), grouping, and place value with whole numbers between 0 and 100.</td>
<td>MA.A.2.1.1.a counts money up to $20.00, using nickels, dimes, quarters, and half-dollars. MA.A.2.1.1.b counts the number of noses in the room; the number of eyes; the number of fingers, etc., in the most efficient manner.</td>
<td>3</td>
</tr>
<tr>
<td>MA.A.2.1.2 uses number patterns and the relationships among counting, grouping, and place value strategies to demonstrate an understanding of the whole number system.</td>
<td>MA.A.2.1.2.a using a hundreds chart, describes as many patterns as possible in 5 minutes. MA.A.2.1.2.b writes the number represented by $2 + 400 + 70$ without adding.</td>
<td>3, 4</td>
<td></td>
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## A. Number Sense, Concepts, and Operations

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<td>Grades 3-5</td>
<td>MA.A.2.2.1</td>
<td><strong>MA.A.2.2.1.a</strong> uses place-value concepts of grouping based upon powers of ten (thousandths, hundredths, tenths, tens, hundreds, thousands) within the decimal number system.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td><strong>MA.A.2.2.1.a</strong> chooses the best estimate below of the number of breaths per day, knowing that, on the average, human beings breathe 984 times each hour.</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>MA.A.2.2.1.b</strong> writes the number represented by 7000 + 3 + 20 + 600, without adding.</td>
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<tr>
<td></td>
<td>MA.A.2.2.2</td>
<td><strong>MA.A.2.2.2.a</strong> recognizes and compares the decimal number system to the structure of other number systems such as the Roman numeral system or bases other than ten.</td>
<td>1, 2, 3, 4, 7, 8, 10</td>
</tr>
<tr>
<td></td>
<td><strong>MA.A.2.2.2.a</strong> compares the characteristics of other number systems (such as Roman, Mayan, Egyptian, base 2, and modular) and, in small groups, develops a new number system and explains how the basic operations work within that system.</td>
<td>1, 2, 3, 4, 7, 10</td>
<td></td>
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<tr>
<td></td>
<td><strong>MA.A.2.2.2.b</strong> extends and explains the pattern of counting as adding one more each time, given the following context. In Cambodia people use a counting system like the following: one, two, three, four, five, five-one, five-two, five-three, five-four, ten, ten-one, ten-two, ten-three, ten-four, ten-five, ten-five-one, ten-five-two, ten-five-three, ten-five-four, twenty.</td>
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### A. Number Sense, Concepts, and Operations

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<td><strong>Grades 6-8</strong></td>
<td>MA.A.2.3.1 understands and uses exponential and scientific notation.</td>
<td>MA.A.2.3.1.a gives two examples of each of the following:</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>a) large numbers containing more than 2 non-zero digits correctly represented in scientific notation (such as distance to planets).</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>b) very small numbers containing more than 2 non-zero digits correctly represented in scientific notation (such as atomic units).</td>
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<td></td>
<td>The student defends why these numbers are best represented in scientific notation and explains what the exponent represents in each.</td>
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<td></td>
<td><strong>Sample Solutions</strong>: 4.56 ( \times 10^{19} )</td>
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<tr>
<td></td>
<td></td>
<td>This number, not represented in scientific notation, would require 20 digits which would make it cumbersome to work with. The exponent 19 means 4.56 multiplied by 1,000,000,000,000,000,000,000,000,000,000,000,000, which gives the equivalent whole number representation.</td>
<td></td>
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## A. Number Sense, Concepts, and Operations

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<tr>
<td>Grades 6-8</td>
<td>MA.A.2.3.2</td>
<td>The student understands the structure of number systems other than the decimal number system.</td>
<td>MA.A.2.3.2.a Given the following situation, determines and draws the five-bar code for the missing digit. In 1963 the U.S. Postal Service began using five-digit zip codes in order to expedite its handling of the mail. In order for the codes to be read by scanners, each digit of our decimal system is represented by five bars. When a five-digit zip code is written, it begins and ends with a single long bar, called a framing bar. The student uses the following zip codes to match the bar codes with the digits 0-9:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>66045</td>
<td>1, 2, 3, 4, 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>02138</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>35487</td>
<td></td>
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Then the student answers the question: Which digit is missing above? The student draws the code for the missing digit and explains the strategy she or he used to find it.
## A. Number Sense, Concepts, and Operations

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<td>Grades 9-12</td>
<td>MA.A.2.4.1 understands and uses the basic concepts of limits and infinity.</td>
<td>MA.A.2.4.1.a graphs the equation $y=2^x$ on a graphing calculator and describes the graph for $x &lt; -10$ and for $x &gt; 10$.</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.A.2.4.2 understands and uses the real number system.</td>
<td>MA.A.2.4.2.a determines the diameter of the largest circular mirror that will fit through a 3 ft. x 8 ft. door.</td>
<td>3, 4</td>
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<tr>
<td></td>
<td></td>
<td>MA.A.2.4.2.b answers questions such as:</td>
<td>2, 3, 4</td>
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<tr>
<td></td>
<td></td>
<td>Is the product of two rational numbers always rational?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Why or why not?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Is the product of two irrational numbers always irrational?</td>
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<td></td>
<td></td>
<td>Why or why not?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MA.A.2.4.3 understands the structure of the complex number system.</td>
<td>MA.A.2.4.3.a describes how to find $i^{1996}$ after exploring the pattern generated below:</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>if $i=\sqrt{-1}$ then find $i^2$, $i^3$, and $i^4$.</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>MA.A.2.4.3.b graphs numbers in the complex plane.</td>
<td>2, 3, 4</td>
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A. Number Sense, Concepts, and Operations

3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

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<td>Grades PreK-2</td>
<td>MA.A.3.1.1 understands and explains the effects of addition and subtraction on whole numbers, including the inverse (opposite) relationship of the two operations.</td>
<td>MA.A.3.1.1.a explains the result of a given situation without computing. Example: A student has seven crayons; a friend gives him three more; he gives three to a friend. How many crayons does the student have now?</td>
<td>2, 3</td>
</tr>
<tr>
<td></td>
<td>MA.A.3.1.2 selects the appropriate operation to solve specific problems involving addition and subtraction of whole numbers.</td>
<td>MA.A.3.1.2.a given a context, selects appropriate operation (+ or -) to put in each box to make a number sentence true. For example, what operations will make the following statement true? (3 \ n \ 4 \ n \ 2 \ n \ 6 = 11)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MA.A.3.1.3 adds and subtracts whole numbers to solve real-world problems, using appropriate methods of computing, such as objects, mental mathematics, paper and pencil, and calculator.</td>
<td>MA.A.3.1.3.a uses a table, chart, or diagram to organize combinations of items to buy for 20 cents and explains how combinations were chosen. Combinations should include at least five combinations of items and all the money should be spent. Bookmarks 4 cents each Pencils 6 cents each Stickers 2 cents each Erasers 7 cents each Pencil toppers 3 cents each Notepads 11 cents each</td>
<td>2, 3, 4, 6</td>
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<td>Grades 3-5</td>
<td>MA.A.3.2.1</td>
<td>Achievements of the benchmarks may be demonstrated when the student understands and explains the effects of addition, subtraction, and multiplication on whole numbers, decimals, and fractions, including mixed numbers, and the effects of division on whole numbers, including the inverse relationship of multiplication and division. MA.A.3.2.1.a given 15 counters, arranges the counters to relate the expressions 15 ÷ 3 = 5 and 3 x 5 = 15. The student then draws a picture to explain the relationship. MA.A.3.2.1.b cooperatively designs a fund-raising activity for the classroom, including the design of a product, an appropriate selling price, calculated cost, income, and profit.</td>
<td>2, 3</td>
</tr>
<tr>
<td>Grades 3-5</td>
<td>MA.A.3.2.2</td>
<td>Solves the following problem: Suppose the class is collecting cans to raise money. On Monday 5 cans are collected. The recycle company gives $.05 per can. How much did the class earn on the first day? If the class collects 7 cans on Tuesday, 12 on Wednesday, and 8 on Thursday, how much money will the class have earned by Friday morning? If the goal is $5.00 per week, has the class reached it? If not, how many more cans must be collected on Friday to reach the goal?</td>
<td>1, 2, 3, 4, 5, 6, 7, 8</td>
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<td>Grades 3-5</td>
<td>MA.A.3.2.3 adds, subtracts, and multiplies whole numbers, decimals, and fractions, including mixed numbers, and divides whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator.</td>
<td>Achievement of the benchmarks may be demonstrated when the student MA.A.3.2.3.a describes number patterns in a given context. For example, the student fills in the chart. Looking at the diagonal list of numbers starting with the product 1 in the top left-hand corner and ending at the product 100 in the bottom right-hand corner, the student writes one true statement about all these products. <strong>Solution:</strong> The numbers are all perfect squares. MA.A.3.2.3.b explains her or his method of computing and the rationale for the computation method, then justifies why she or he spent money the way she or he did in a given situation. <strong>Example</strong> Students work in groups to spend $100.00 cost effectively by choosing at least 5 items from newspapers or catalogs.</td>
<td>2, 3, 4, 6, 8</td>
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A. **Number Sense, Concepts, and Operations**

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<td>Grades 6-8</td>
<td>MA.A.3.3.1 understands and explains the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, including mixed numbers, and decimals, including the inverse relationships of positive and negative numbers.</td>
<td>MA.A.3.3.1.a uses the formula $P=1.2 \frac{W}{H^2}$, where $P =$ pounds per square inch $W =$ your weight in pounds $H =$ width of heel in inches, to describe the effect on the pounds of pressure exerted when $H$ increases or decreases.</td>
<td>2, 3, 4</td>
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| Grades 6-8 | MA.A.3.3.2 selects the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers, ratios, proportions, and percents, including the appropriate application of the algebraic order of operations. | MA.A.3.3.2.a performs mathematical operations on the numbers 2, 4, 6, 8, and 10 to form today’s date. The student finds either the day, the month and day, or the month, day, and year. To find the date each number (2, 4, 6, 8, and 10) must be used, but can only be used once. For example, March 23, 1995, might be solved by finding 23, 323, or 32395, using the correct order of operations. **Sample Solution:**  
23 = 10 x 8 ÷ 4 + 6 ÷ 2  
80 ÷ 4 + 3  
20 + 3  
23 | 3, 4 |
| | | MA.A.3.3.2.b defends the correct application of the algebraic order of operations in a contextual situation. **Example** Mike and Martha want to put a fence around their yard. They know the formula for the perimeter of a rectangle is “two times the length plus the width.” The yard is 180 feet long and 200 feet wide. Mike says they need 560 feet of fencing, saying “2 times 180 is 360 and 360 plus 200 is 560”. Martha disagrees, saying they need 760 feet of fencing. The student pretends to be in Martha’s place, and defends her answer. | 2, 3, 4 |
A. **Number Sense, Concepts, and Operations**

3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

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<tbody>
<tr>
<td><strong>Grades 6-8</strong></td>
<td>MA.A.3.3.3 adds, subtracts, multiplies, and divides whole numbers, decimals, and fractions, including mixed numbers, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator.</td>
<td>MA.A.3.3.3.a uses the advertisement section of a newspaper as a resource to write a descriptive plan to complete a shopping trip in a contextual situation. <strong>Example:</strong> The student has a holiday budget of $150 and 6 family members to buy gifts for. The student’s mother will leave him or her at the mall at 12:00 noon and pick him or her up at 5:30 PM. Given this situation, the student includes a schedule showing the maximum time that should be allotted for finding each gift and a purchase plan with costs that include a 6% sales tax.</td>
</tr>
</tbody>
</table>

| Goal 3 Standards | 1, 2, 3, 4, 5, 6 |
A. Number Sense, Concepts, and Operations

3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

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<tr>
<td>Grades 9-12</td>
<td>MA.A.3.4.1 understands and explains the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships.</td>
<td>MA.A.3.4.1.a finds solutions for the following scenario: Anthony and James are twins who are planning their 30 year retirement plan. James, a fun-loving type, wants to spend $2000 per year for the first 10 years on a vacation, and then save $2000 per year for 20 years at 5% interest compounded annually for a total investment of $40,000 plus interest. Anthony, the more cautious twin, plans to invest $2000 per year at 5% interest compounded annually for the first 10 years and then spend $2000 per year on vacations for the last 20 years, for a total investment of $20,000 plus interest. Which twin will have the larger retirement fund at the end of 30 years? The student explains his or her answer.</td>
<td>2, 3, 4, 6, 7</td>
</tr>
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</table>
A. **Number Sense, Concepts, and Operations**

3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

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<tr>
<td>Grades 9-12</td>
<td>MA.A.3.4.2 selects and justifies alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems.</td>
<td>MA.A.3.4.2.a uses and identifies appropriate properties to explain a procedure that could be used to mentally compute a problem such as ( \left( \frac{5}{3} \times 800 \right) \times 6 ). <strong>Sample Solutions:</strong> The student applies the commutative property to reorder the numbers inside the parenthesis and then the associative property to regroup the numbers, so that the problem becomes ( 800 \times \left( \frac{5}{3} \times 6 \right) ) which simplifies to ( 800 \times 10 ), which is 8000.</td>
<td>2, 3</td>
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</table>
## A. Number Sense, Concepts, and Operations

3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

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<tr>
<td>Grades 9-12</td>
<td>MA.A.3.4.2 (continued) selects and justifies alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems.</td>
<td>MA.A.3.4.2.b describes the capture/recapture method used by ecologists and zoologists to estimate the number of animals in the wild. (Some animals are captured, tagged, and returned to the wild. Several samples are taken and the number of tagged animals in the sample is compared to the total population size.) Through a similar process of tagging and sampling, the student estimates the population of “bears” in a classroom forest, which has been constructed by another student. When the estimation is complete, each student identifies the advantages and disadvantages of the capture/recapture technique.</td>
<td>1, 2, 3, 4, 6, 7, 8</td>
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### A. Number Sense, Concepts, and Operations

3. The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

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<tr>
<td>Grades 9-12</td>
<td>MA.A.3.4.3 adds, subtracts, multiplies, and divides real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator.</td>
<td>MA.A.3.4.3.a explains who was correct and why, in the following scenario: Andrea was helping Jeffrey with his homework. The problem was $\frac{x}{7} + \frac{x}{7} =$ . Jeffrey had gotten $\frac{x}{7}$ . Andrea was sure he was wrong because she had gotten $2 \frac{x}{7}$. Lamanda overheard Jeffrey and Andrea arguing over the correct response. She told them they were both wrong because her denominator was 12. The student answers the questions: Who was correct? Why?</td>
<td>2, 3, 4, 8, 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA.A.3.4.3.b fills out a 1040 tax return form and analyzes the components of the form. The analysis should include a discussion of the relationship of income, family size, and expenses to the amount of tax. The student discusses ways to avoid owing more money at the end of the next tax year.</td>
<td>2, 3, 4, 8, 9</td>
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</table>
4. The student uses estimation in problem solving and computation.

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<tr>
<td>Grades</td>
<td>PreK-2</td>
<td>MA.A.4.1.1 provides and justifies estimates for real-world quantities. MA.A.4.1.1.a estimates sizes of quantities and explains the estimation given the following context: The student looks at three piles of buttons. There are 12 buttons in the middle pile. The student guesses how many more buttons there are in the biggest pile and how many fewer buttons there are in the smallest pile. The student explains her or his guess.</td>
<td>2, 3</td>
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</tbody>
</table>
### A. Number Sense, Concepts, and Operations

4. The student uses estimation in problem solving and computation.

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<td>Grades 3-5</td>
<td>MA.A.4.2.1 uses and justifies different estimation strategies in a real-world problem situation and determines the reasonableness of results of calculations in a given problem situation.</td>
<td>MA.A.4.2.1.a estimates the total on a cash register tape and checks real total for accuracy. MA.A.4.2.1.b determines the reasonableness of results of calculations in the following problem situation: Leticia bought a milk shake at the ice cream shop and gave the clerk a $10 bill. She got $9.61 in change. The student explains whether this amount of change is reasonable.</td>
</tr>
</tbody>
</table>
A. Number Sense, Concepts, and Operations

4. The student uses estimation in problem solving and computation.

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<td>Grades 6-8</td>
<td>MA.A.4.3.1</td>
<td>Uses estimation strategies to predict results and to check the reasonableness of results.</td>
<td>2, 3</td>
</tr>
<tr>
<td></td>
<td>MA.A.4.3.1.a</td>
<td>Uses rounding and concepts of common percents to estimate real quantities. The student estimates given the following information: The average cost of housing in the Florida panhandle is 54.5% of the cost of housing in central Florida, and the average cost of housing in central Florida is $127,500. The student explains his or her answer.</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Solution: Because 54.5% is close to 50%, housing in the panhandle area would be close to half the cost in central Florida (rounded to $128,000), or approximately $64,000.</td>
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### A. Number Sense, Concepts, and Operations

4. The student uses estimation in problem solving and computation.

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<tr>
<td>Grades 9-12</td>
<td>MA.A.4.4.1 uses estimation strategies in complex situations to predict results and to check the reasonableness of results.</td>
<td>Achievement of the benchmarks may be demonstrated when the student makes reasonable predictions using estimation strategies to get a sensible answer to a real-world situation. For example, an experienced painter can paint the exterior of a house in 5 hours. An inexperienced helper would take 9 hours for the same house. Given that the two painters are working together, the student chooses the most sensible answer to the question, how long will it take them to paint the house? The student explains his or her decision. a. 45 hours d. 6 hours b. 14 hours e. 1 hour c. 10 hours f. 3 hours</td>
<td>1, 3, 4</td>
</tr>
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</table>

**Solution:**
Two painters together should be faster than either of them alone. Therefore only answers e and f are possible solutions. Since the inexperienced painter takes 9 hours alone, he will probably not decrease the time sufficiently enough to finish in only one hour. Answer f is most realistic.
A. Number Sense, Concepts, and Operations

5. The student understands and applies theories related to numbers.

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<tr>
<td>Grades PreK-2</td>
<td>MA.A.5.1.1 clasifies and models numbers as even or odd.</td>
<td>MA.A.5.1.1.a builds a model to show that 12 is an even number.</td>
<td>2, 3, 4</td>
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<tr>
<td></td>
<td></td>
<td>Sample Solution: Two even rows of 12 cookies show 12 is an even number of cookies.</td>
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A. Number Sense, Concepts, and Operations

5. The student understands and applies theories related to numbers.

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</table>
| Grades 3-5 | MA.A.5.2.1 understands and applies basic number theory concepts, including primes, composites, factors, and multiples. | MA.A.5.2.1.a determines the common multiples of 2 and 3 from a drawing or model of a contextual situation. **Example:** A fence is to be built and bushes planted across the back of a lot that is 24 meters in length. Posts are placed every three meters. Green bushes are to be planted every two meters except in front of posts. In front of these posts, rose bushes are to be planted. The student draws a diagram to show the location of the posts, the green bushes, and the rose bushes. The student labels all the numbers and answers the question: What number relationships do the roses represent? **Sample Solution:**

```
post bush rose
```

Common multiples of 2 and 3 will be all the places that have roses. | 2, 3, 4 |
### A. Number Sense, Concepts, and Operations

5. The student understands and applies theories related to numbers.

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<td>Grades 6-8</td>
<td>MA.A.5.3.1 uses concepts about numbers, including primes, factors, and multiples, to build number sequences.</td>
<td>MA.A.5.3.1.a uses a model to justify common multiples.</td>
<td>2, 3, 4</td>
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<td></td>
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<td><strong>Example:</strong></td>
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<td>A double strand of blinking holiday lights has a strand of red lights blinking every 9 seconds and a strand of green lights blinking every 15 seconds. The student determines after how many seconds both strands will be on at the same time and justifies his or her answer.</td>
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<td></td>
<td></td>
<td><strong>Sample Solution:</strong></td>
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<tr>
<td></td>
<td></td>
<td>red—on at 9, 18, 27, 36, <strong>45</strong>, 54 seconds</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>green—on at 15, 30, <strong>45</strong>, 60, 75, 90 seconds</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>They will both be on at <strong>45</strong> seconds, the least common multiple of 9 and 15.</td>
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## A. Number Sense, Concepts, and Operations

5. The student understands and applies theories related to numbers.

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<td>Grades 9-12</td>
<td>MA.A.5.4.1</td>
<td>The student applies special number relationships such as sequences and series to real-world problems.</td>
<td>MA.A.5.4.1.a determines the value of an investment over a period of time using a formula, given the following context: In 1626, Peter Minuit purchased Manhattan Island for $24 worth of beads, cloth, and trinkets. If the Indians had received the $24 in cash and invested it in a savings account paying 6% per year interest compounded annually, the student determines how much would they have on deposit after 5 years and after 10 years. The student uses a spreadsheet to determine the value of the $24 investment in 1995.</td>
</tr>
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</table>

**Solution:**

Using the formula for compound interest $A = P(1 + r)^t$, the amount would be $A = 24(1.06)^{369}$, or $A = 24(2177029343)$, which gives $52,248,704,232$. 


B. Measurement

1. The student measures quantities in the real world and uses the measures to solve problems.

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<tr>
<td>Grades PreK-2</td>
<td>MA.B.1.1.1.1 uses and describes basic measurement concepts, including length, weight, digital and analog time, temperature, and capacity.</td>
<td>MA.B.1.1.1.a works in a small group to select five objects in the classroom, identify four standard measurements and four student-made measuring tools, and then measures the objects and records and compares the measurements.</td>
<td>1, 2, 3, 4, 8</td>
</tr>
<tr>
<td></td>
<td>MA.B.1.1.2 uses standard customary and metric (centimeter, inch) and nonstandard units, such as links or blocks, in measuring real quantities.</td>
<td>MA.B.1.1.2.a uses links to measure height of a student, length of a table, distance around the head of a student. The student measures the same objects with a tape measure and compares results.</td>
<td>3</td>
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B. Measurement

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<td>Grades 3-5</td>
<td>MA.B.1.2.1 uses concrete and graphic models to develop procedures for solving problems related to measurement including length, weight, time, temperature, perimeter, area, volume, and angle.</td>
<td>MA.B.1.2.1.a draws a variety of squares and rectangles on graph paper and constructs a table that records the number of boxes along the length and the width of each square and rectangle in order to discover any patterns for finding area and perimeter. MA.B.1.2.1.b given a small group of tangrams with the area of the square equal to one whole unit, identifies the area of the other size tangram pieces and the total area of all the pieces provided. Solution: square=1 unit 2 small triangles=1/2 unit each medium triangle=1 unit 2 large triangles=2 units each parallelogram=1 unit Total area=8 units</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>Grades 3-5</td>
<td>MA.B.1.2.2 solves real-world problems involving length, weight, perimeter, area, capacity, volume, time, temperature, and angles.</td>
<td>MA.B.1.2.2.a uses cereal (or any dry pourable substitute) and a variety of boxes to determine the relationship between the dimensions of a box and the capacity of the box.</td>
<td>3</td>
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B. Measurement

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<td>Grades 6-8</td>
<td>MA.B.1.3.1 uses concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids and cylinders.</td>
<td>MA.B.1.3.1.a folds and cuts graph paper to build two-dimensional shapes. The student then compares the area and perimeter of triangles, squares, and trapezoids that have the same base length and height and documents these findings on a table with a written interpretation of the findings. With tape the student then builds three-dimensional solids using the two-dimensional models as faces. The student predicts the surface areas, and then tests predictions. Using graph paper as a guide, the student estimates the volume of each model. Working with a group, the student tests estimations and contributes to a group consensus on a working formula for finding the volume of the three-dimensional models.</td>
<td>2, 3, 4, 8, 9</td>
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### B. Measurement

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<td>Grades</td>
<td>MA.B.1.3.2 uses concrete and graphic models to derive formulas for finding rates, distance, time, and angle measures.</td>
<td>MA.B.1.3.2.a uses a cut out triangle to write the formula for the sum of the interior angles of a triangle. <strong>Example</strong>: The student cuts a triangle from a sheet of paper. The student cuts each of the three angles from the triangle and lays them so that they form adjacent angles. The student compares results with results from other class members to get a rule that applies to all triangles.</td>
<td>2, 3, 4, 8</td>
</tr>
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### B. Measurement

1. The student measures quantities in the real world and uses the measures to solve problems.

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<td>Grades 6-8</td>
<td>MA.B.1.3.3 understands and describes how the change of a figure in such dimensions as length, width, height, or radius affects its other measurements such as perimeter, area, surface area, and volume.</td>
<td>MA.B.1.3.3.a determines and justifies comparable pricing for different size pizza. <strong>Example:</strong> The eighth grade is having a pizza sale. They have 2 sizes: 6-inch diameter and 12-inch diameter. A 6-inch pizza sells for $2.75. The student determines the fair price for a 12-inch pizza and justifies the answer.</td>
<td>2, 3, 4, 8, 9</td>
</tr>
<tr>
<td></td>
<td>MA.B.1.3.4 constructs, interprets, and uses scale drawings such as those based on number lines and maps to solve real-world problems.</td>
<td>MA.A.1.3.4.a scales a picture from a coloring book or greeting card by drawing a 2-cm by 2-cm grid on the picture. The student creates a 1-cm by 1-cm grid on plain paper and a 3-cm by 3-cm grid on a legal size manila folder. The student then duplicates the original picture one square at a time. A “key” showing the scale should be included.</td>
<td>1, 2, 3, 4</td>
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1. The student measures quantities in the real world and uses the measures to solve problems.

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<td><strong>Grades 9-12</strong></td>
<td>MA.B.1.4.1 uses concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cylinders, cones, and pyramids.</td>
<td>MA.B.1.4.1.a working with cans and string of various sizes, measures the distance around the cans (circumference) and the distance across the cans (diameter). The student then makes a chart with columns listing circumference, diameter, and circumference divided by diameter. The latter column should be close to 3 (or $\pi$). The student then uses the columns to find the formula $C=\pi d$. MA.B.1.4.1.b uses the label cut from a can and flattened to develop a formula for the surface area of a cylinder.</td>
<td>2, 3, 4</td>
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B. Measurement

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<td>Grades 9-12</td>
<td>MA.B.1.4.2 uses concrete and graphic models to derive formulas for finding rate, distance, time, angle measures, and arc lengths.</td>
<td>MA.B.1.4.2.a uses a computer graphics program to explore relationships between arc lengths and angle measures, makes a conjecture about that relationship, then verifies the conjecture through comparison with classmates. <strong>Example:</strong> Using a computer graphics program, the student draws a circle with an angle whose vertex is on the circle (inscribed angle). The student uses the program capabilities to measure both the angle and the intercepted arc length. Then, dragging one of the points where the angle and circle intersect, changing the intercepted arc length, the student observes what happens to the angle measure as the arc changes. The student then makes a conjecture about the relationship of an inscribed angle and its intercepted arc length and writes out the rule.</td>
<td>2, 3, 4, 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA.B.1.4.2.b draws a curve on a piece of graph paper. The student explains how to find the approximate area and discusses how to find a closer approximation of the area.</td>
<td>2, 3, 4</td>
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1. The student measures quantities in the real world and uses the measures to solve problems.

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<td>Grades 9-12</td>
<td>MA.B.1.4.3 relates the concepts of measurement to similarity and proportionality in real-world situations.</td>
<td>MA.B.1.4.3.a uses ratio and proportion to find distances that are difficult to measure directly.</td>
<td>3</td>
</tr>
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</table>

**Example:** Surveyors often need to know distances that are difficult to measure directly, like the distance across a canyon or a lake. Given the following picture, the student finds the distance X across the canyon.

![Diagram](canyon_diagram.png)
### B. Measurement

2. The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).

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<td>Grades PreK-2</td>
<td>MA.B.2.1.1 uses direct (measured) and indirect (not measured) comparisons to order objects according to some measurable characteristics (length, weight).</td>
<td>MA.B.2.1.1.a writes a class story about things in the classroom that are long, things that are heavy, etc.</td>
<td>1, 2, 3, 4, 8</td>
</tr>
<tr>
<td></td>
<td>MA.B.2.1.2 understands the need for a uniform unit of measure to communicate in real-world situations.</td>
<td>MA.B.2.1.2.a explores the need for consistent units of measure by first determining and recording how many steps it takes to cross the room. The student compares that answer with the answers from other students in the classroom. The student then explains why there might be different answers and how a single answer might be found.</td>
<td>2, 3, 4, 8</td>
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## B. Measurement

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<tr>
<td><strong>Grades 3-5</strong></td>
<td>MA.B.2.2.1 uses direct (measured) and indirect (not measured) measures to calculate and compare measurable characteristics.</td>
<td>MA.B.2.2.1.a identifies, graphs, and compares characteristics of local trees to such special trees as the world’s tallest tree, the coastal redwood, the tree with the largest seed, the coconut palm, and other selected trees. Then the student uses data to calculate approximate diameters of the trees not found in this area.</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td></td>
<td>MA.B.2.2.2 selects and uses appropriate standard and nonstandard units of measurement, according to type and size.</td>
<td>MA.B.2.2.2.a given the choice between a yardstick and a ruler, chooses the appropriate tool to measure the height of a door. MA.B.2.2.2.b uses a town map to write directions from the town library to the classroom and to figure the estimated time it will take to travel from the library to the classroom.</td>
<td>3</td>
</tr>
</tbody>
</table>
## B. Measurement

2. The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).

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<td><strong>Grades 6-8</strong></td>
<td>MA.B.2.3.1 uses direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units.</td>
<td>MA.B.2.3.1.a selects appropriateness of direct or indirect measurement for given situation. <strong>Example</strong> The student determines whether placing an order for a living room carpet would require a direct or indirect measurement and explains why. The student then calculates the number of square yards of carpet needed if the room is 12 feet by 15 feet.</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.B.2.3.2 solves problems involving units of measure and converts answers to a larger or smaller unit within either the metric or customary system.</td>
<td>MA.B.2.3.2.a computes reaction time in seconds, given the speed of a ball in miles per hour. <strong>Example</strong> The pitcher of the high school baseball team has been clocked throwing the ball at 70 miles per hour. The distance from the pitcher’s mound to home plate is 60 feet 6 inches. The student determines, for that speed and that distance, how many seconds the batter has to react.</td>
<td>3, 4</td>
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B. Measurement

2. The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).

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<td>Grades 9-12</td>
<td>MA.B.2.4.1 selects and uses direct (measured) and indirect (not measured) methods of measurement as appropriate.</td>
<td>MA.B.2.4.1.a draws all rectangles with whole number dimensions whose area is 24 square feet. The student prepares an argument to convince the class that all rectangles with whole number dimensions have been drawn, and determines which of the rectangles has the largest perimeter. MA.B.2.4.1.b describes a method for measuring the diameter of the moon using a dime.</td>
<td>2, 3, 4, 9</td>
</tr>
<tr>
<td></td>
<td>MA.B.2.4.2 solves real-world problems involving rated measures (miles per hour, feet per second).</td>
<td>MA.B.2.4.2.a determines how many seconds it takes the bungee jumper to fall 200 feet if the rate of fall is about 120 mi/hr.</td>
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### B. Measurement

3. The student estimates measurements in real-world problem situations.

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<tr>
<td>Grades PreK-2</td>
<td>MA.B.3.1.1 using a variety of strategies, estimates lengths, widths, time intervals, and money and compares them to actual measurements.</td>
<td>MA.B.3.1.1.a estimates the length of the room. The student describes the procedure used to make the estimate, then measures the room to check the estimate.</td>
<td>2, 3</td>
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</table>
### B. Measurement

3. The student estimates measurements in real-world problem situations.

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<td>Grades 3-5</td>
<td>MA.B.3.2.1 solves real-world problems involving estimates of measurements, including length, time, weight, temperature, money, perimeter, area, and volume.</td>
<td>MA.B.3.2.1.a uses concrete examples, such as various-sized boxes, in a cooperative group to estimate how many of certain-sized objects will fit inside the boxes. The student then helps fill the boxes with the objects and evaluates the group’s results.</td>
<td>1, 2, 3, 4, 8</td>
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### B. Measurement

3. The student estimates measurements in real-world problem situations.

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<tr>
<td>6-8</td>
<td>MA.B.3.3.1 solves real-world and mathematical problems involving estimates of measurements including length, time, weight/mass, temperature, money, perimeter, area, and volume in either customary or metric units.</td>
<td>MA.B.3.3.1.a constructs a line graph depicting the energy output of a typical middle school student over a 24-hour period, which includes a school day and an afternoon soccer practice. The student labels the y-axis energy output in estimated calories and the x-axis time in hours. With the graph the student provides a written description of the activities.</td>
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B. Measurement

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<tr>
<td>Grades 9-12</td>
<td>MA.B.3.4.1 solves real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimates the effects of measurement errors on calculations.</td>
<td>MA.B.3.4.1.a given that donuts cost $3.59 a dozen and donut holes are $2.29 for 30, the student in a small group compares the price of a donut to a donut hole to determine the better buy, based on price per cubic inch of each. The student explains the process used to approximate the volume. Then the student uses the price per cubic inch of the donut hole to re-price the donut and the price of the donut per cubic inch to re-price the donut hole. The student discusses the effect on prices of using an “estimated” volume.</td>
<td>2, 3, 4, 6</td>
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B. Measurement

4. The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

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<td>Grades PreK-2</td>
<td>MA.B.4.1.1 selects and uses an object to serve as a unit of measure, such as a paper clip, eraser, or marble.</td>
<td>MA.B.4.1.1.a given a variety of objects, decides which would be the most accurate unit to use for a given situation.</td>
<td>3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.B.4.1.2 selects and uses appropriate instruments and technology, such as scales, rulers, and clocks, to measure within customary or metric systems.</td>
<td>MA.B.4.1.2.a decides which would be the most appropriate tool to use for the expected measurement when given a variety of objects. <strong>Example:</strong> The student uses a 12” ruler to determine the length of a table to the nearest foot.</td>
<td>1, 2, 3, 4</td>
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### B. Measurement

4. The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

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<td><strong>Grades 3-5</strong></td>
<td>MA.B.4.2.1</td>
<td>The student determines which units of measurement, such as seconds, square inches, and dollars per tankful, to use with answers to real-world problems.</td>
<td>MA.B.4.2.1.a Given a variety of capacity measurement examples, identifies the most appropriate unit of measurement. Example: The student identifies the most appropriate unit of measurement for the amount of water in a swimming pool, a dose of medicine, or an amount of orange juice in party punch. 3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.B.4.2.2</td>
<td>The student selects and uses appropriate instruments and technology, including scales, rulers, thermometers, measuring cups, protractors, and gauges to measure in real-world situations.</td>
<td>MA.B.4.2.2.a Determines capacity and effective instruments for measuring capacity given a context. Example: The student’s class has decided to participate in the collect-a-million-pennies project. Before collecting pennies, the student must decide how much space the pennies will fill if they are stored in gallon milk jugs and how much they will all weigh to determine where they can be stored. The student writes a report of the findings, including recommendations for different size storage containers. 2, 3, 4, 8</td>
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## B. Measurement

4. The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

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<td>Grades 6-8</td>
<td>MA.B.4.3.1 selects appropriate units of measurement and determines and applies significant digits in a real-world context. (Significant digits should relate to both instrument precision and to the least precise unit of measurement.)</td>
<td>MA.B.4.3.1.a given a list of measurements, identifies the appropriate measurement for each defined example. <strong>Example:</strong> The student determines which of the following measures would be most appropriate for each of the described situations. 500 yd 461.6 cm 462 ft 460 m a) The length of a photo to be framed. b) The feet of fencing needed for the back yard. c) The distance to grandma’s house. d) The cloth needed to make costumes for the play. MA.B.4.3.1.b Finds the perimeter of the following figure. The student discusses how precise the perimeter is, and why.</td>
<td>3, 4, 2, 3, 4</td>
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### B. Measurement

4. The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

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<td>Grades 6-8</td>
<td>MA.B.4.3.1 (continued) selects appropriate units of measurement and determines and applies significant digits in a real-world context. (Significant digits should relate to both instrument precision and to the least precise unit of measurement.)</td>
<td>MA.B.4.3.1.c chooses appropriate graduated cylinder for precision of measurement required. <strong>Example</strong> The student determines which of the following graduated cylinders would be best to accurately measure 4.23 ml and explains why.</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td></td>
<td>MA.B.4.3.2 selects and uses appropriate instruments, technology, and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation.</td>
<td>MA.B.4.3.2.a in a cooperative group, estimates the cost of a construction job, given the job's blueprints, specifications, material costs and labor. (Teachers can request samples of this information from construction contractors. Students could be required to research the material and labor costs. The cooperative groups could participate in a bid process with discussion including why group estimates vary.)</td>
<td>1, 2, 3, 4, 6, 8, 9</td>
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### B. Measurement

4. The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

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<td>Grades 9-12</td>
<td>MA.B.4.4.1 determines the level of accuracy and precision, including absolute and relative errors or tolerance, required in real-world measurement situations.</td>
<td>MA.B.4.4.1.a determines tolerance in a given context. <strong>Example:</strong> A machinist is making reinforcement bars for concrete from metal rods. The rods should be 1.75 centimeters in diameter (w=diameter). The student determines the allowable error (tolerance), given the specifications require the interval $1.70 \leq w \leq 1.80$. The student then explains how he or she arrived at the answer.</td>
<td>2, 3</td>
</tr>
<tr>
<td></td>
<td>MA.B.4.4.2 selects and uses appropriate instruments, technology, and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation.</td>
<td>MA.B.4.4.2.a compares the accuracy in measuring percentage of body fat using a caliper rule versus full immersion in water. The student researches advantages and disadvantages of each method, making a table that shows instances where one method is more appropriate than the other.</td>
<td>1, 2, 3, 4, 7, 8</td>
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</table>
# C. Geometry and Spatial Sense

1. The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.

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<td>Grades PreK-2</td>
<td>MA.C.1.1.1 understands and describes the characteristics of basic two- and three-dimensional shapes.</td>
<td>achievement of the benchmarks may be demonstrated when the student MA.C.1.1.1.a describes a cereal box from memory. MA.C.1.1.1.b makes and describes the following figures on a geoboard: a) a 5-sided figure b) the smallest possible triangle c) a large triangle d) a square The student then writes a sentence beside each picture to describe it.</td>
<td>2, 3, 4</td>
</tr>
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</table>
1. The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.

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<td>Grades 3-5</td>
<td>MA.C.1.2.1 given a verbal description, draws and/or models two- and three-dimensional shapes and uses appropriate geometric vocabulary to write a description of a figure or a picture composed of geometric figures.</td>
<td>MA.C.1.2.1.a compares and contrasts squares and rectangles using characteristics such as number of sides, size of angles, length of sides. MA.C.1.2.1.b describes what a trash can would look like to a fly on the ceiling.</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>The student</td>
<td>Achievement of the benchmarks may be demonstrated when the student</td>
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### C. Geometry and Spatial Sense

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<td>Grades 6-8</td>
<td>MA.C.1.3.1 understands the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in two and three dimensions.</td>
<td>MA.C.1.3.1.a given a variety of regular polygons (triangle, square, pentagon, hexagon, etc.), investigates the relationship between the number of sides and the number of diagonals of any regular polygon. The student justifies the relationship and supports the conclusions.</td>
<td>2, 3, 4, 9</td>
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## C. Geometry and Spatial Sense

1. The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.

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<td>Grades 9-12</td>
<td>MA.C.1.4.1 uses properties and relationships of geometric shapes to construct formal and informal proofs.</td>
<td>MA.C.1.4.1.a cuts the following into four pieces of identical size and shape and proves they are congruent.</td>
<td>2, 3, 4</td>
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[Diagram of a geometric shape]
C. Geometry and Spatial Sense

2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.

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<td>Grades PreK-2</td>
<td>MA.C.2.1.1 understands basic concepts of spatial relationships, symmetry, and reflections.</td>
<td>MA.C.2.1.1.a completes the other side of the picture of the tree.</td>
<td>3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.C.2.1.2 uses objects to perform geometric transformations, including flips, slides, and turns.</td>
<td>MA.C.2.1.2.a traces cut-out geometric figures and flips, sides, or turns the traced figure and compares it to the original.</td>
<td>3, 4</td>
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C. Geometry and Spatial Sense

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<td>Grades 3-5</td>
<td>MA.C.2.2.1 understands the concepts of spatial relationships, symmetry, reflections, congruency, and similarity.</td>
<td>MA.C.2.2.1.a makes a tessellation using two different pattern blocks. (A tessellation is a covering or tiling of a plane with no gaps or overlaps.) MA.C.2.2.1.b makes a class quilt with a theme.</td>
<td>3, 4</td>
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<tr>
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<td></td>
<td></td>
<td>2, 3, 4, 8</td>
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### C. Geometry and Spatial Sense

2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.

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<td>Grades 3-5</td>
<td>MA.C.2.2.2</td>
<td>Achievement of the benchmarks may be demonstrated when the student selects the appropriate figure that is the result of turning or rotating a given figure.</td>
<td>3, 4</td>
</tr>
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<td></td>
<td>MA.C.2.2.2.a</td>
<td>describes and draws the two new shapes that result from dividing a given figure. The student selects and circles the figure {a, b, c, or d} that could be the same drawing in a new position.</td>
<td></td>
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<tr>
<td></td>
<td>MA.C.2.2.2.b</td>
<td>The student cuts a 3 cm x 4 cm rectangle in half on the diagonal and describes and draws the two new shapes. The student first determines if they are congruent, and then determines how many different shapes can be made if the two triangles are pieced together so that one pair of congruent sides is shared. With a partner, the student describes the shapes and also writes about them, noting the new characteristics.</td>
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C. Geometry and Spatial Sense

2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.

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<td>Grades</td>
<td>MA.C.2.3.1 understands the geometric concepts of symmetry, reflections, congruency, similarity, perpendicularly, parallelism, and transformations, including flips, slides, turns, and enlargements.</td>
<td>MA.C.2.3.1.a makes a square from post-it notes and draws a three color design. The student then explores the many designs that can be created by flipping, sliding, and turning one or more of the post-it notes. The student posts a favorite design and uses geometric terms to describe how it was created and what is being shown by combining the 4 pieces.</td>
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C. Geometry and Spatial Sense

2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.

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<td>Grades 6-8</td>
<td>MA.C.2.3.2 predicts and verifies patterns involving tessellations (a covering of a plane with congruent copies of the same pattern with no holes and no overlaps, like floor tiles).</td>
<td>Achievement of the benchmarks may be demonstrated when the student MA.C.2.3.2.a decides which figures can be tessellated. <strong>Example:</strong> Of an equilateral triangle, a square, a regular pentagon, and a regular hexagon, the student determines and explains which can be fundamental regions for a tessellation and which cannot. MA.C.2.3.2.b researches tessellations, including those of Escher and Islamic artists. In a group, the student designs a unique carpet for the classroom that uses tessellations. MA.C.2.3.2.c defines a common real-world 2-dimensional shape (this could be a picture of an object) that has characteristics that allow it to be tessellated. The student draws an example of a tessellation using this shape.</td>
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C. **Geometry and Spatial Sense**

2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.

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<td>Grades 9-12</td>
<td>MA.C.2.4.1 understands geometric concepts such as perpendicularity, parallelism, tangency, congruency, similarity, reflections, symmetry, and transformations including flips, slides, turns, enlargements, rotations, and fractals.</td>
<td>MA.C.2.4.1.a constructs a Koch snowflake (fractal) by dividing the sides of an equilateral triangle into thirds and constructing another equilateral triangle on the central third of each side. This process is repeated on every remaining straight line segment. The student compares the changes in both the perimeter and the area. (Computer graphics programs can help a great deal!)</td>
<td>2, 3, 4, 7, 9</td>
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- **Step 1**
- **Step 2**
- **Step 3**
C. **Geometry and Spatial Sense**

2. The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.

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<td><strong>Grades 9-12</strong></td>
<td>MA.C.2.4.2 analyzes and applies geometric relationships involving planar cross-sections (the intersection of a plane and a three-dimensional figure).</td>
<td>MA.C.2.4.2.a decides when a planar cross-section is possible. <strong>Example</strong>: The student considers where it is possible for a planar cross-section of a cube to be a hexagon. The student draws a cube to demonstrate how this could happen. The student then determines which of the following is also possible with a planar cross-section of a cube: a pentagon; a triangle; a quadrilateral that is not a parallelogram. The student justifies the answer. MA.C.2.4.2.b uses a computer paint program or geometry supposer to create a shape and to show the result of various transformations.</td>
<td>2, 3, 4</td>
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**C. Geometry and Spatial Sense**

3. The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.

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<td>Achievement of the benchmarks may be demonstrated when the student</td>
<td>2, 3</td>
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<td></td>
<td><strong>MA.C.3.1.1</strong> uses real-life experiences and physical materials to describe, classify, compare, and sort geometric figures, including squares, rectangles, triangles, circles, cubes, rectangular solids, spheres, pyramids, cylinders, and prisms, according to the number of faces, edges, bases, and corners.</td>
<td>MA.C.3.1.1.a given real objects in geometric shapes, such as cereal boxes for rectangular solids, classifies them according to specified criteria and explains why she or he grouped them as she or he did.</td>
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<td></td>
<td><strong>MA.C.3.1.2</strong> plots and identifies positive whole numbers on a number line.</td>
<td>MA.C.3.1.2.a figures out what 2-digit number the teacher has chosen by asking questions such as: Does it come before 25? Does it come after 30? The student uses a number line drawn on the chalk board to illustrate the process.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Sample Solution:</strong></td>
<td>The process of narrowing the interval to reach the correct number causes him or her to “order” many numbers. For example, if the teacher’s number is 47, guesses might have to include many numbers, such as 20, 75, 50, 30, 40, 45, and 46 before arriving at 47.</td>
<td></td>
</tr>
</tbody>
</table>
3. The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.

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<tbody>
<tr>
<td>Grades 3-5</td>
<td>MA.C.3.2.1 represents and applies a variety of strategies and geometric properties and formulas for two- and three-dimensional shapes to solve real-world and mathematical problems.</td>
<td>MA.C.3.2.1.a using graph paper, shows what happens to the perimeter and area of a square when the length of the side is doubled. The student writes a “formula” to show the change.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MA.C.3.2.2 identifies and plots positive ordered pairs (whole numbers) in a rectangular coordinate system (graph).</td>
<td>MA.C.3.2.2.a plays “Guess My Rule.” The student makes up a rule that generates one number from another, (for example, “add 5” generates 8 from 3). Makes a table of both numbers from her or his rule. The student plots the points on a graph and describes the graph.</td>
<td>1, 2, 3, 4</td>
</tr>
</tbody>
</table>
### C. Geometry and Spatial Sense

3. The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.

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<tbody>
<tr>
<td>Grades 6-8</td>
<td>MA.C.3.3.1 represents and applies geometric properties and relationships to solve real-world and mathematical problems.</td>
<td>MA.C.3.3.1.a solves a real-world problem given a context.</td>
<td>2, 3, 4, 5, 9</td>
</tr>
</tbody>
</table>

**Example:** The local convenience store sells an average of 275 soft drinks each day. Over the course of one year, the manager noticed an imbalance in the income from soft drinks. Based on the information below, the student determines which drink is priced incorrectly (A, B, C, or D) and justifies the answer. Volume = $\frac{1}{3} \pi r^2 h$; use 3.14 for $\pi$.

<table>
<thead>
<tr>
<th>radius (cm)</th>
<th>height (cm)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) 5</td>
<td>10</td>
<td>$0.24$</td>
</tr>
<tr>
<td>B) 7</td>
<td>12</td>
<td>$0.72$</td>
</tr>
<tr>
<td>C) 8</td>
<td>15</td>
<td>$0.92$</td>
</tr>
<tr>
<td>D) 9</td>
<td>18</td>
<td>$1.39$</td>
</tr>
</tbody>
</table>
### C. Geometry and Spatial Sense

3. The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.

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<tbody>
<tr>
<td>Grades</td>
<td>The student</td>
<td>Achievement of the benchmarks may be demonstrated when the student</td>
<td>2, 3, 4, 5, 9</td>
</tr>
<tr>
<td>6-8</td>
<td>MA.C.3.3.1 (continued) represents and applies geometric properties and relationships to solve real-world and mathematical problems.</td>
<td>MA.C.3.3.1.b applies formulas for the area of one figure to approximate the area of another figure.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Example:</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>What is the best approximation for the area of a circle?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ r^2, 2r^2, 3r^2, 4r^2 ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The student justifies his or her answer.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MA.C.3.3.2 identifies and plots ordered pairs in all four quadrants of a rectangular coordinate system (graph) and applies simple properties of lines.</td>
<td>MA.C.3.3.2.a given an index card with an ordered pair on it as he or she enters the room, the student seats him or herself as though the desks were a rectangular coordinate system. The student then generates a list of the names of all students who represent different lines. For example: a vertical line, ( y = 5 ); ( x + y = 4 ); ( x - y &lt; 6 ); and so on.</td>
<td>2, 3, 4, 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA.C.3.3.2.b explains to another student how to get from the classroom she or he is in to another location on the school grounds using symbols and Cartesian maps.</td>
<td>2, 3, 8</td>
</tr>
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### C. Geometry and Spatial Sense

3. The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.

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<tr>
<td>Grades 9-12</td>
<td>MA.C.3.4.1 represents and applies geometric properties and relationships to solve real-world and mathematical problems including ratio, proportion, and properties of right triangle trigonometry.</td>
<td>MA.C.3.4.1.a applies right triangular trigonometry to solve a problem. <strong>Example</strong>: The windows of Ingrid’s house are to be built so that the eaves completely shade them from the sun in the summer and allow full sun in the winter. The eaves have an overhang of 3.5 feet. The sun in midwinter is at an angle of elevation of 25(^\circ) and the sun in midsummer is at an angle of elevation of 70(^\circ). The student describes the required location and length of the window.</td>
<td>2, 3, 6, 8</td>
</tr>
<tr>
<td></td>
<td>MA.C.3.4.2 using a rectangular coordinate system (graph), applies and algebraically verifies properties of two- and three-dimensional figures, including distance, midpoint, slope, parallelism, and perpendicularity.</td>
<td>MA.C.3.4.2.a describes the classroom in terms of a coordinate system. The student assigns variable coordinates to the corners, such as (a,0), (a,b), (0,a). The student then uses these coordinates to find quantities such as the length of the wall or the midpoint of the floor. The student also uses these to prove all cases of parallelism and perpendicularity and verifies by measuring.</td>
<td>3</td>
</tr>
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D. Algebraic Thinking

1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.

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<td>Grades PreK-2</td>
<td>MA.D.1.1.1 describes a wide variety of classification schemes and patterns related to physical characteristics and sensory attributes, such as rhythm, sound, shapes, colors, numbers, similar objects, and similar events.</td>
<td>MA.D.1.1.1.a describes the pattern below. slslns1nslnslnslnslns</td>
<td>2, 3</td>
</tr>
<tr>
<td></td>
<td>MA.D.1.1.2 recognizes, extends, generalizes, and creates a wide variety of patterns and relationships using symbols and objects.</td>
<td>MA.D.1.1.2.a draws the next figure following the pattern and determines how many small squares are needed each time to extend the pattern below.</td>
<td>2, 3, 4</td>
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## D. Algebraic Thinking

1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.

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<td>Grades 3-5</td>
<td>MA.D.1.2.1 describes a wide variety of patterns and relationships through models, such as manipulatives, tables, graphs, and rules using algebraic symbols.</td>
<td>MA.D.1.2.1.a decides which of the graphs most likely shows the number of pockets that each child had on a given day. The student explains why he or she chose that graph and not the others, given the following information. There are 20 students in Mr. Arnett's class. On Tuesday most of the students in the class said they had pockets in the clothes they were wearing.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Goal 3 Standards</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1, 2, 3, 4, 7</td>
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**Solution:**

1, 3, 6, 10, 15, 21, 28, 36, 47, 57
D. Algebraic Thinking

1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.

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<td>Grades 3-5</td>
<td>MA.D.1.2.2 generalizes a pattern, relation, or function to explain how a change in one quantity results in a change in another.</td>
<td>Achieves the benchmarks may be demonstrated when the student</td>
<td>3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.D.1.2.2.a</td>
<td>Chooses one of three different situations by looking at the pattern in each option.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Example: Mr. Trumpet would like to offer you a job. He will hire you for ten days. He will pay you one of three ways: a) $1 the first day, $2 the second day, $3 the third day and so on. b) $0.10 the first day, $0.20 the second day, and each day after will be double the day before. c) $6 a day for each of the ten days.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Which way would you choose? Why?</td>
<td></td>
<td></td>
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D. Algebraic Thinking

1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.

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<td>Grades 6-8</td>
<td>MA.D.1.3.1 describes a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, graphs, expressions, equations, and inequalities.</td>
<td>MA.D.1.3.1.a answers the following questions:</td>
<td>2, 3, 4</td>
</tr>
</tbody>
</table>
|        |          | $\begin{array}{|c|c|} \hline X & Y \\ \hline 1 & 5 \\ 3 & 9 \\ 7 & 17 \\ 8 & a \\ 16 & b \\ c & 63 \\ d & 23 \\ e & 35 \\ \hline \end{array}$ | 1. What is being done to the numbers in the X column to get the numbers in the Y column?  
2. Describe the pattern you would use to find the numbers for “a” and “b.”  
3. Use the pattern described in number 2 to find the numbers for “c,” “d,” and “e.”  
4. Describe how you found X when you used your pattern in question number 3. | |
|        | MA.D.1.3.2 creates and interprets tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships. | MA.D.1.3.2.a graphs and explains the growth of a population over time of a colony of organisms that doubles once a day. | 1, 2, 3, 4, 7, 8 |
D. Algebraic Thinking

1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.

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<td>Grades 9-12</td>
<td>MA.D.1.4.1 describes, analyzes, and generalizes relationships, patterns, and functions using words, symbols, variables, tables, and graphs.</td>
<td>MA.D.1.4.1.a analyzes graphs. Example: The human body uses energy, measured in calories, at varying rates according to the level of the activity. The body burns more calories during physical exercise than when at rest. A person’s size, physical condition, metabolic rate, and other factors make calorie consumption very individual. When describing calorie consumption for a particular activity, average calorie consumption is used. In a local newspaper an article claimed that walking requires the same amount of energy as dancing. The claim was supported by the following graphs: Due to a printer's error, the titles, labels, and scales were unreadable.</td>
<td>1, 2, 3, 4, 5, 8, 9</td>
</tr>
<tr>
<td></td>
<td>The student</td>
<td>Achievement of the benchmarks may be demonstrated when the student</td>
<td></td>
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1. The student completes the graphs in a manner that supports the claim of the article.
2. The student completes the graphs in a manner that is more consistent with true calorie consumption.
3. The student then writes a description of the two versions of the graphs in a letter to the editor about the error in the article.
D. **Algebraic Thinking**

1. The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.

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| Grades 9-12 | MA.D.1.4.2 determines the impact when changing parameters of given functions. | MA.D.1.4.2.a investigates the impact on a function when a constant is changed:  
  a) Compares the cost of purchasing a $75,000 home at 6% interest over 15 years and 30 years.  
  b) Compares the cost of purchasing a $75,000 home over 30 years at 6% and 8% interest. | 1, 2, 3, 4, 7, 8  |
D. Algebraic Thinking

2. The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.

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| Grades PreK-2 | MA.D.2.1.1 understands that geometric symbols (m, n) can be used to represent unknown quantities in expressions, equations, and inequalities. | MA.D.2.1.1.a fills in a variety of number sentences such as \(2 + s = 6\). The student writes a word problem that the sentence could represent. MA.D.2.1.1.b selects a number from a list to make a number sentence true. Example: The student puts one of the following numbers in the box to make the sentence true. \[ n + 2 > 5 \]
|        |           |                                 | 2, 3, 4          |
|        | MA.D.2.1.2 uses informal methods to solve real-world problems requiring simple equations that contain one variable. | MA.D.2.1.2.a draws a picture to represent: “Six students would like to each have an apple. If they have 3 apples, how many more do they need?” \[3 + n = 6\] | 3          |
D. Algebraic Thinking

2. The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.

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<td>Grades 3-5</td>
<td>MA.D.2.2.1 represents a given simple problem situation using diagrams, models, and symbolic expressions translated from verbal phrases, or verbal phrases translated from symbolic expressions, etc.</td>
<td>MA.D.2.2.1.a writes a sentence to describe all the possibilities for the cube and the cone in the problem below. Andre the Seal likes to do tricks. He can balance 19 ounces on his nose. His favorite trick is to balance a ball, a cube, and a cone on his nose. The ball weighs 8 ounces. In an organized way, show how much the cube and the cone could weigh.</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.D.2.2.2 uses informal methods, such as physical models and graphs, to solve real-world problems involving equations and inequalities.</td>
<td>MA.D.2.2.2.a uses words and pictures to show that Mario could be right in the following scenario. Mario ate 1/2 of a pizza. Celia ate 1/2 of another pizza. Mario said he ate more pizza than Celia, but Celia said they both ate the same amount.</td>
<td>2, 3, 4</td>
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## D. Algebraic Thinking

2. The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.

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<td>Grades 6-8</td>
<td>MA.D.2.3.1 represents and solves real-world problems graphically, with algebraic expressions, equations, and inequalities.</td>
<td>MA.D.2.3.1.a writes an expression that would describe a book that is overdue by 10 days; by 15 days; by “d” days. The school library overdue book fine is as follows: First week............................$1.00 After first week.....$1.00 + $.25 a day</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MA.D.2.3.2 uses algebraic problem-solving strategies to solve real-world problems involving linear equations and inequalities.</td>
<td>MA.D.2.3.2.a describes the strategy or strategies used to answer the following question. Suppose there are 45 animals on the beach, some turtles and some pelicans. There are 104 legs on the beach. How many turtles and how many pelicans are on the beach?</td>
<td>2, 3, 4</td>
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## D. Algebraic Thinking

2. The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.

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| Grades 9-12 | **MA.D.2.4.1** represents real-world problem situations using finite graphs, matrices, sequences, series, and recursive relations. | MA.D.2.4.1.a given the following information: according to the Rand McNally Road Atlas, the population of Florida was about 13 million in 1990 and the average annual growth rate was 3.3%.  
  a) Writes a recursive formula for the sequence of the annual population growth in Florida.  
  b) Estimates the population in the year 2010.  
  c) Discusses reasons why this estimate might not be correct—why would the growth rate not remain constant? | 2, 3, 4, 7, 8 |
|       | **MA.D.2.4.2** uses systems of equations and inequalities to solve real-world problems graphically, algebraically, and with matrices. | MA.D.2.4.2.a explains problem-solving strategy in detail within a context. Example: An enterprising resort owner ran the following offer: Two-week vacations at the following rates:  
  $75 per day for sunny days  
  $50 per day for rainy days  

  At the end of Greg’s vacation, his resort bill was $950 for the two weeks. The student determines how many days it rained during Greg’s vacation and explains his or her response. | 2, 3, 4 |
E. Data Analysis and Probability

1. The student understands and uses the tools of data analysis for managing information.

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<td>MA.E.1.1.1</td>
<td>Achievement of the benchmarks may be demonstrated when the student</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>displays solutions to problems by generating, collecting, organizing, and analyzing data using simple graphs and charts.</td>
<td>MA.E.1.1.1.a stacks colored cubes (blocks) on the table to show whether classmates rode the bus, rode in a car, rode a bicycle, or walked to school today. The student discusses his or her “graph.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MA.E.1.1.2</td>
<td>displays data in a simple model to use the concepts of range, median, and mode.</td>
<td>1, 2, 3, 4, 7, 8</td>
</tr>
<tr>
<td></td>
<td>MA.E.1.1.2.a</td>
<td>informally explores range and central tendency. Example: After collecting data from classmates, the student places an “X” over the number line to indicate the total number of brothers and sisters the students in the class have. The student discusses the results.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sample Solution:</td>
<td>In a class of 11 students, the results might be:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA.E.1.1.2.b</td>
<td>uses concrete objects to explore median. Example: The student has three bookshelves. There are 8 books on one shelf, 4 books on one shelf, and 6 books on one shelf. The student uses blocks to show how many books should be put on a shelf in order to have the same number on each shelf.</td>
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## E. Data Analysis and Probability

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<td>Grades PreK-2</td>
<td>MA.E.1.1.3 analyzes real-world data by surveying a sample space and predicting the generalization onto a larger population through the use of appropriate technology, including calculators and computers.</td>
<td>MA.E.1.1.3.a takes a class survey and records results in a chart and/or pictograph. The student makes a prediction of school-wide responses to the same survey using calculators to facilitate working with large numbers.</td>
<td>1, 2, 3, 4, 7, 8</td>
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E. Data Analysis and Probability

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<td>Grades 3-5</td>
<td>MA.E.1.2.1 solves problems by generating, collecting, organizing, displaying, and analyzing data using histograms, bar graphs, circle graphs, line graphs, pictographs, and charts.</td>
<td>MA.E.1.2.1.a collects and graphs data from an experiment to determine if the amount of light affects plant growth. The student then summarizes the findings.</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>MA.E.1.2.2 determines range, mean, median, and mode from sets of data.</td>
<td>MA.E.1.2.2.a demonstrates an understanding of concepts of central tendency by answering the following: Find the average number of visits to the dentist this class has had from October to February. (calculator may be used.)</td>
<td>2, 3, 4</td>
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E. Data Analysis and Probability

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<td>Grades 3-5</td>
<td>MA.E.1.2.3 analyses real-world data to recognize patterns and relationships of the measures of central tendency using tables, charts, histograms, bar graphs, line graphs, pictographs, and circle graphs generated by appropriate technology, including calculators and computers.</td>
<td>MA.E.1.2.3.a uses computer graphing software to analyze sets of data (favorite ice cream flavor, TV show, vacation activity, etc.) and to recognize patterns and relationships of the measures of central tendency using tables, charts, histograms, line graphs, pictographs, and circle graphs.</td>
<td>2, 3, 4, 7, 8</td>
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E. Data Analysis and Probability

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<td>Grades 6-8</td>
<td>MA.E.1.3.1 collects, organizes, and displays data in a variety of forms, including tables, line graphs, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations.</td>
<td>MA.E.1.3.1.a participates in a class census. The student picks from a list of topics such as favorite type of music, favorite fast food eatery, favorite professional sports team, etc. (Ideally there are as many topics as students in class.) The student then asks fellow students in class about the topic of his or her census. Example: The student asks other students to name their favorite fast food eatery, and records responses in an organized table. The student then represents the data in graph form and presents it to the class with a written interpretation of what the graph shows about the class.</td>
<td>1, 2, 3, 4, 6, 7, 9</td>
</tr>
<tr>
<td></td>
<td>MA.E.1.3.2 understands and applies the concepts of range and central tendency (mean, median, and mode).</td>
<td>MA.E.1.3.2.a collects temperatures every 30 minutes throughout a 12 hour period beginning at 8:00 AM and ending at 8:00 PM. The student then finds the measures of central tendency and writes an argument that defends which measure best describes what the day was like or that states that none of the measures of central tendency alone fairly describe the temperature for the day.</td>
<td>1, 2, 3, 4, 6, 8, 9</td>
</tr>
</tbody>
</table>
### E. Data Analysis and Probability

1. The student understands and uses the tools of data analysis for managing information.

<table>
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<tr>
<td>Grades 6-8</td>
<td>MA.E.1.3.3 analyzes real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers.</td>
<td>MA.E.1.3.3.a analyzes and makes predictions from collected data using calculators to apply formulas for measures of central tendency, and organizes data in the form of charts, tables, or graphs. The student uses computer graphing software to organize collected data in a quality display. Example: The student makes a quality display of the data on salaries from a company (from President to Custodian). First the student finds the mean, median, and mode, then prepares a discussion of why these numbers accurately or inaccurately present the measures of central tendency. The student also shows how eliminating the “extremes” affects the measures of central tendency.</td>
<td>2, 3, 4, 7, 8</td>
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### E. Data Analysis and Probability

1. The student understands and uses the tools of data analysis for managing information.

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<tr>
<td>Grades 9-12</td>
<td>MA.E.1.4.1 Interprets data that has been collected, organized, and displayed in charts, tables, and plots.</td>
<td>MA.E.1.4.1.a selects types of stocks. The student collects data about the stock, such as the price of the stock and the number of shares sold, over a period of time. The student graphs data, and uses the graph to compare and contrast different stocks.</td>
<td>1, 2, 3, 4, 6, 7, 8</td>
</tr>
<tr>
<td></td>
<td>MA.E.1.4.2 Calculates measures of central tendency (mean, median, and mode) and dispersion (range, standard deviation, and variance) for complex sets of data and determines the most meaningful measure to describe the data.</td>
<td>MA.E.1.4.2.a prepares a statement describing the effect of a 3.8% raise to all employees on both the mean and standard deviation of both expenses and profit. MA.E.1.4.2.b determines the most meaningful measure of central tendency for a given situation.</td>
<td>2, 3, 4</td>
</tr>
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</table>

**Example:** The student has six test grades for each of six weeks. The student determines which measures of central tendency that he or she would want the teacher to use to find the grade for the class, and explains why.
E. Data Analysis and Probability

1. The student understands and uses the tools of data analysis for managing information.

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<tr>
<td>Grades 9-12</td>
<td>MA.E.1.4.3 analyzes real-world data and makes predictions of larger populations by applying formulas to calculate measures of central tendency and dispersion using the sample population data and using appropriate technology, including calculators and computers.</td>
<td>MA.E.1.4.3.a constructs spreadsheets on graphing calculators or computers using real-world sample data sets. The student makes predictions of larger populations by applying formulas in the spreadsheets to calculate measures of central tendencies and dispersion using the sample population data.</td>
<td>2, 3, 4, 7, 8</td>
</tr>
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</table>

Example: The student collects data from class members on height and speed in athletics. Using a graphing calculator to check correlation, the student determines whether the relationship is linear, exponential, logarithmic, or power. The student then prepares a display to convince coaches that height should or should not be a consideration in picking teams.

Example: The student collects data on average income for men and for women over the last 20 years. The student examines relationships such as (time, gender), (time, difference), (time, ratio) etc. The student then finds the best-fit correlations that are relevant and prepares a presentation of the findings, conclusions, and predictions.
## E. Data Analysis and Probability

2. The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.

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<tr>
<td>Grades PreK-2</td>
<td>MA.E.2.1.1 understands basic concepts of chance and probability.</td>
<td>MA.E.2.1.1.a using terms such as probably, likely, not possible, etc., describes chance events such as weather.</td>
<td>2, 4</td>
</tr>
<tr>
<td></td>
<td>MA.E.2.1.2 predicts which simple event is more likely, equally likely, or less likely to occur.</td>
<td>MA.E.2.1.2.a makes predictions about the likelihood of simple events. <strong>Example:</strong> Ten yellow pieces of candy and two green pieces of candy are placed in a bag. The student determines which color would most likely show up if one piece of candy were taken out of the bag at random. The student explains the answer.</td>
<td>2, 3, 4</td>
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</tbody>
</table>
**E. Data Analysis and Probability**

2. The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.

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</table>
| Grades 3-5 | MA.E.2.2.1 uses models, such as tree diagrams, to display possible outcomes and to predict events. | MA.E.2.2.1.a uses models to determine how many ways three children can be arranged on three chairs.  
MA.E.2.2.1.b determines and expresses the probability of an event happening (any arrangement) as a ratio in fraction form. | 2, 3, 4         |
|       | MA.E.2.2.2 predicts the likelihood of simple events occurring.            | MA.E.2.2.2.a drawing from a set of six blue beads and three gold beads, uses ratios to state the likelihood of each color being drawn; conducts experiments to test predictions. | 2, 3, 4         |
### E. Data Analysis and Probability

2. The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.

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<tr>
<td>Grades 6-8</td>
<td>MA.E.2.3.1</td>
<td>MA.E.2.3.1.a compares experimental results with mathematical expectations of probabilities.</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>The student compares experimental results with mathematical expectations of probabilities.</td>
<td>MA.E.2.3.1.a compares experimental results with mathematical expectations of probabilities in a context.</td>
<td>Goal 3 Standards</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two marked coins are tossed in the air at the same time. The student computes the mathematical probability of both coins landing with heads up. Then the student tosses the two coins 20 times and records the results each time. The student determines whether the experimental result matches the mathematical expectation and explains why it does or does not. If it does not, the student explains whether the experiment could be changed to get a better match.</td>
<td>2, 3, 4</td>
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<td></td>
<td></td>
<td>In a small group, the student helps to estimate the percentage of the earth’s surface covered by each of the continents and by each of the major oceans, with other land and other water as additional categories. The student tosses and catches an inflatable globe 100 times, with the position of the index finger on the dominant hand recorded. Using statistical data for surface area of the earth and area of each land and water body listed, the student computes the percentage of the earth covered by each land or water body. Then the student compares experimental probability to both estimation and mathematical expectations.</td>
<td>2, 3, 4</td>
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<td>Grades 6-8</td>
<td>MA.E.2.3.2 determines the odds for and the odds against a given situation.</td>
<td>MA.E.2.3.2.a after determining all possible outcomes when tossing two different colored number cubes numbered 1-6, determines the odds for and against tossing the cubes and having the sum of the two numbers shown equal 6.</td>
<td>2, 3, 4</td>
</tr>
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2. The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.

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<tr>
<td>Grades 9-12</td>
<td>MA.E.2.4.1 determines probabilities using counting procedures, tables, tree diagrams, and formulas for permutations and combinations.</td>
<td>Achievement of the benchmarks may be demonstrated when the student uses tree diagrams to organize data for possible outcomes with real world data.</td>
<td>2, 3, 4</td>
</tr>
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</table>

**Example:** The class is going to order pizza for a celebration of success at the end of the year. The student determines how many possible combinations there are given the choice of shape, crust, and 3 toppings:

- round thin crust pepperoni sausage mushroom
- square deep dish onion bacon hamburger anchovies cheese green pepper
## E. Data Analysis and Probability

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<tr>
<td>Grades 9-12</td>
<td>MA.E.2.4.2</td>
<td>determines the probability for simple and compound events as well as independent and dependent events.</td>
<td>2, 3, 4</td>
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<tr>
<td></td>
<td>MA.E.2.4.2.a</td>
<td>determines probability within a context.</td>
<td></td>
</tr>
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<td></td>
<td><strong>Example:</strong> The pictures below are dart targets. For a competition the student gets to choose which one to use. The object of the competition is to see how many darts can be landed in a shaded region. The student determines which gives the greatest probability of hitting a shaded area and explains the choice.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MA.E.2.4.2.b</td>
<td>paints, in imagination or actually, a four-inch cube using a favorite color. The student cuts it into 64 one-inch cubes and selects one of them at random. Then the student predicts the probability that if it is tossed like a die, the top face of the one-inch cube will be painted the chosen color.</td>
<td>2, 3, 4</td>
</tr>
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</table>
### E. Data Analysis and Probability

3. The student uses statistical methods to make inferences and valid arguments about real-world situations.

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<td><strong>Grades PreK-2</strong></td>
<td>MA.E.3.1.1 designs a simple experiment to answer a class question, collects appropriate information, and interprets the results using graphical displays of information, such as line graphs, pictographs, and charts.</td>
<td>MA.E.3.1.1.a designs an experiment to predict which is the top selling cereal. The student makes a bar graph to show the kinds of cereal classmates ate for breakfast. The student then makes conclusions from the graph.</td>
</tr>
<tr>
<td></td>
<td>MA.E.3.1.2 decides what information is appropriate and how data can be collected, displayed, and interpreted to answer relevant questions.</td>
<td>MA.E.3.1.2.a goes to the cafeteria and counts how many students eat all their lunch and how many do not. The student discusses any problems he or she found in collecting the data.</td>
</tr>
</tbody>
</table>

**Goal 3 Standards**

1, 2, 3, 4, 8
E. Data Analysis and Probability

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<tr>
<td>Grades 3-5</td>
<td>MA.E.3.2.1 designs experiments to answer class or personal questions, collects information, and interprets the results using statistics (range, mean, median, and mode) and pictographs, charts, bar graphs, circle graphs, and line graphs.</td>
<td>MA.E.3.2.1.a from the graph of the number of cars that come to a complete stop at a four-way-stop intersection, decides if it can be determined which stop more often: Big cars or small cars? Cars going straight or turning? Cars with children or those without? The student explains why some might stop more often.</td>
<td>1, 2, 3, 4, 8</td>
</tr>
<tr>
<td></td>
<td>MA.E.3.2.2 uses statistical data about life situations to make predictions and justifies reasoning.</td>
<td>MA.E.3.2.2.a predicts sales of a CD or the price of an electronic game for teenagers based on statistical data.</td>
<td>1, 2, 3, 4, 6, 7, 8</td>
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## E. Data Analysis and Probability

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<tr>
<td><strong>Grades 6-8</strong></td>
<td>MA.E.3.3.1 formulates hypotheses, designs experiments, collects and interprets data, and evaluates hypotheses by making inferences and drawing conclusions based on statistics (range, mean, median, and mode) and tables, graphs, and charts.</td>
<td>MA.E.3.3.1.a designs, implements, and monitors a water conservation plan. The student collects data on daily water usage, before and after the plan has been implemented, and writes a paper justifying the plan.</td>
<td>1, 2, 3, 4, 8, 9</td>
</tr>
<tr>
<td></td>
<td>MA.E.3.3.2 identifies the common uses and misuses of probability and statistical analysis in the everyday world.</td>
<td>MA.E.3.3.2.a stages a debate to confirm or disprove the claims of nationally advertised products.</td>
<td>1, 2, 3, 4, 6, 8, 9</td>
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3. The student uses statistical methods to make inferences and valid arguments about real-world situations.

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<tr>
<td>Grades 9-12</td>
<td>MA.E.3.4.1 designs and performs real-world statistical experiments that involve more than one variable, then analyzes results and reports findings.</td>
<td>MA.E.3.4.1.a designs an experiment, collects the data, and makes a presentation to address a real problem. Example: A new radio station is interested in opening in the student’s area. The radio station’s owners have commissioned the student to tell them whether or not this area needs a station like theirs. The student designs an experiment, collects the data, and makes a presentation that shows there is a need for this particular station in town.</td>
<td>1, 2, 3, 4, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td></td>
<td>MA.E.3.4.2 explains the limitations of using statistical techniques and data in making inferences and valid arguments.</td>
<td>MA.E.3.4.2.a finds the latest population report for her or his city and explains a) whether or not this information is an accurate count of the population or a close estimate; and b) some assumptions one should make in interpreting the number.</td>
<td>1, 2, 3, 4, 6, 7, 8, 9, 10</td>
</tr>
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</table>
Chapter 4: Teaching and Learning

Chapter Highlights

- The Unique Perspective of Mathematics
- New Approaches to Teaching and Learning
- Instructional Strategies for the 21st Century
- Infusing a Multicultural Perspective
- Snapshot of an Effective Mathematics Classroom
- Teaching Diverse Students
  - Diverse Needs
  - Developmental Differences
  - Learning Preferences
  - Students With Disabilities
  - Students Who Are Limited English Proficient
  - At-Risk Students
- Putting These Ideas to Work

The Unique Perspective of Mathematics

To reach the goal of developing mathematical power for all students requires the design of a mathematical program that is likely to be different from current practices. This framework articulates a vision of teaching and learning that changes not only what we teach but how we teach. This vision is based on the goals of the National Council of Teachers of Mathematics (NCTM) which emphasize that all students should experience mathematics in such a way that they

- learn to reason mathematically in their own lives;
- learn to value mathematics as a useful, creative environment;
- learn to use mathematics as a form of communication;
- become confident of their mathematical abilities; and
- become mathematical problem solvers.
As students plan, invent, design, and evaluate their own ideas and problems, they see the power of mathematics in revealing the significant patterns and relationships around them. They experience how mathematical rules are connected. They also learn the value of mathematics in describing and modeling real-world phenomena, its uses in other disciplines, and the advantages of mathematical language in communicating complex thoughts and information. Mathematically powerful students look for patterns, relationships, and connections as they search for sense and order from complex situations. Their self-confidence grows and develops as they consider alternative approaches, test hypotheses, and select from a variety of tools and models. Through varied problem-solving experiences and investigations, students grow in their ability to use higher level thinking processes and build a greater depth of mathematical understanding.

Problem solving is an integral part of everyday life and should be the central focus of a mathematics program. Mathematically powerful students approach problem solving with confidence and perseverance. They have the ability to solve problems using a variety of mathematical concepts, tools, and techniques. In addition, they can use mathematical language with facility, communicating their own thinking about complex situations through pictures, graphs, diagrams, words, symbols, and numerical examples. The mathematics curriculum should provide numerous opportunities for students to verbalize their ideas, reasoning processes, and opinions. Mathematical power involves the ability and freedom to explore, conjecture, validate, and convince others of solutions and decisions. If an emphasis on reasoning permeates all mathematical activity, eventually students will appreciate the pervasive use and power of reasoning as part of mathematics (NCTM, 1989). In a risk-free environment, students improve their problem-solving skills, gain confidence in doing mathematics, and develop inquiring and persevering minds.

New Approaches to Teaching and Learning

Florida’s education reform initiative calls on educators to redesign their instructional programs so that every student achieves high academic standards. This redesign may include the structure and context of the learning environment and the use of materials, equipment, and resources. School and district leaders must encourage change and look for creative approaches to teaching and learning. Sequencing of courses may be altered; mathematics instruction may be integrated with other areas
of the curriculum; schools and communities may form partnerships; classrooms may be modified to include community settings, museums, nature centers, and other cultural institutions; and electronic networks may link students and teachers across America and to other countries.

Learning theories and instructional practices can inform these new approaches. A tremendous amount of research is available to educators on how children learn and on how to design effective learning environments. This chapter highlights key elements that can help educators, through further investigation, collaborative consideration, implementation, and evaluation, to develop the best learning environments for their unique students.

**Developing a Learning-Centered, Authentic Environment**

Attempts to improve mathematics teaching must be based on an understanding of how students learn. Learning is a natural process of discovering and constructing meaning from information and experience, filtered through the learner’s unique perceptions, thoughts, feelings, and beliefs. The learner grapples with new knowledge until it makes sense and fits into his or her world of understanding.

Based on this knowledge of the learning process, educators are encouraged to design mathematics curricula that allow students to encounter ideas, events, and materials in real-world contexts. Children learn most effectively when actively involved in a subject rather than just hearing or reading about it. Classrooms that are limited to the exclusive use of textbooks, lectures, and paper-and-pencil tasks do not tend to be as successful as those that actively engage students in the learning process. Curiosity, creativity, and higher order thinking are stimulated when experiences are based on real, complex, and relevant ideas and materials. This
immersion in direct experience should be balanced with opportunities for learners to reflect, discuss, and connect concepts with what they have felt, thought, and learned.

Identifying students’ interests and questions also helps engage students in the learning process by stimulating the natural curiosity that students bring to school. Children learn best when called upon to make choices and assume more responsibility for their own learning, while the teacher provides support and guidance.

Some of the most efficient learning occurs when students are collaborating with each other in pairs or small groups. Providing students with the opportunity to interact with others in a variety of settings can enhance knowledge and understanding. Feedback from fellow students can help students clarify areas of understanding as well as misconceptions and questions. Collaborative work can also encourage students to take intellectual risks. Students might pose their own problems, devise their own approaches to problem solving, clarify and defend their conclusions, explore possibilities, and use the results to make informed decisions. Students learn the valuable skill of working effectively with others to solve problems and perform investigations, a skill that will be useful in the workplace and in many other areas of their lives.

Providing a Supportive Environment

The teacher is key to creating a supportive, effective learning environment. Teachers provide this kind of environment when they maintain fair, consistent, and caring policies that respect the individuality of students and focus on individual achievement and cooperative teamwork. Students’ learning is enhanced when others see their potential, genuinely appreciate their unique talents, and accept them as individuals. In such an environment, students can learn the skills of being responsible for themselves, making decisions, working cooperatively, negotiating conflicts, and taking risks; students also have more freedom to do quality work on their own initiative. In addition, a teacher who creates a supportive environment for students can reduce the negative effect of factors that can interfere with learning, such as low self-esteem; lack of self-control; lack of personal goals; expectations of failure or limited success; and feelings of anxiety, insecurity, or pressure. A supportive learning environment and a variety of teaching strategies that promote exploration, discussion, and collaborative learning will help ensure that all children...
have the opportunity to see themselves as capable students, successful in learning mathematics.

**Teachers, Tasks, and Discourse**

This framework envisions teachers who are proficient in

- selecting mathematical tasks to engage students’ interest and intellect;
- providing opportunities to deepen students’ understanding of mathematics and its applications;
- orchestrating classroom discourse to promote investigation and growth;
- using technology and other tools to support students’ mathematical investigations;
- assisting students to seek connections between existing knowledge and new concepts; and
- guiding individual, small-group, and whole class work.

**Selecting Mathematical Tasks**

Good mathematical tasks are those that

- are based on important mathematical ideas, concepts, processes, and perspectives;
- are based on awareness of students’ knowledge, interests, and experiences;
- take into account the different ways students learn mathematics;
- encourage students to make connections and develop a coherent framework for mathematical ideas;
- promotes the skills of problem formulation, problem solving, and mathematical reasoning; and
- represent mathematics as an evolving human activity with daily applications in people’s lives.

**Orchestrating Classroom Discourse**

One of the key components of the classroom of the future is fostering mathematical discourse that is student oriented. The central role of the teacher is to facilitate this discourse so that it contributes to and enhances students’ understanding of
mathematics. To encourage students’ reasoning about mathematics and to involve
them in higher order thinking processes, teachers must be adept at posing clarifying
and provocative questions. Teachers of mathematics can orchestrate discourse by

- posing questions and tasks that elicit, engage, and challenge student’s
  thinking;
- listening carefully to students’ ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students
  bring up during a discussion;
- deciding when and how to attach mathematical notation and language
  to students’ ideas;
- deciding when to provide information, when to clarify an issue, when to
  model, when to lead, and when to let a student struggle with a difficulty; and
- monitoring students’ participation in discussions and deciding when
  and how to encourage each student to participate.

Teachers of mathematics can promote discourse in which students

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and
  communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- prove the conjecture true and try to find an example that would prove
  it false
- try to convince themselves and one another of the validity of particular
  representations, solutions, conjectures, and answers; and
- rely on mathematical evidence and argument to determine validity.

**Instructional Strategies for the 21st Century**

In each mathematics classroom, there is a diverse pool of talent and potential. The
challenge is to structure the learning environment so that each student has the
freedom to use his or her unique strengths to learn or perform, yet be urged, inspired,
and motivated to reach high academic standards. Because all children do not learn in
the same way and have varying backgrounds and experiences, flexible and innovative approaches are needed.

To support innovative mathematics classrooms, the instructional strategies on the following pages are provided as examples of the many kinds of strategies that educators might use as they work toward providing the most useful and engaging educational experiences possible. After further investigation, teachers may use these and other instructional strategies for independent or group work. They can creatively adapt and refine them to best fit the needs of the students and the instructional plan, perhaps incorporating several of these strategies into a single lesson or using them in collaboration with a colleague.
**COOPERATIVE LEARNING:** A strategy in which students work together in small groups to achieve a common goal. Cooperative learning involves more than simply putting students into work or study groups. Teachers promote individual responsibility and positive group interdependence by making sure that each group member is responsible for a given task. Cooperative learning can be enhanced when group members have diverse abilities and backgrounds.

<table>
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<tr>
<th><strong>HOW DO YOU USE IT?</strong></th>
<th><strong>WHAT ARE THE BENEFITS?</strong></th>
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| After organizing students into groups, the teacher thoroughly explains a task to be accomplished within a time frame. The teacher facilitates the selection of individual roles within the group and monitors the groups, intervening only when necessary, to support students working together successfully and accomplishing the task. | • fosters interdependence and pursuit of mutual goals and rewards  
• develops communication and leadership skills  
• increases the participation of shy students  
• produces higher levels of student achievement, thus increasing self-esteem  
• fosters respect for diverse abilities and perspectives |

There are numerous cooperative learning strategies that educators can use to enhance student learning. One of these, Jigsawing, is offered on the next page.
SEQUENCE OF ACTIVITIES: A systematic transition for developing understanding by progressing from an introduction of new ideas through the use of concrete manipulatives to an application of the concept using pictures, graphs, diagrams, or numerical representations, and ending with the versatile representation available by using symbols.
**USE OF MANIPULATIVES:** The use of objects that can be touched and moved by students to introduce or reinforce a concept through observation of mathematical concepts in action.

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<th><strong>HOW DO YOU USE IT?</strong></th>
<th><strong>WHAT ARE THE BENEFITS?</strong></th>
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</table>
| Provide concrete objects to introduce a new concept by allowing students to explore the meaning of the concept in a visual style that can vary to meet the needs of the individual students. | • helps students build conceptual understanding  
• promotes active learning  
• builds motivation and positive attitudes towards mathematics  
• can be adapted to meet diverse student needs  
• encourages direct observation of mathematical patterns, procedures, and relationships |

**DRILL AND PRACTICE ACTIVITIES:** Activities are used to promote quick recall. Some examples of “drill and practice” activities are skip counting, the number line, triangle flash cards, chalkboard drills, and basic fact games.

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<td>Students practice algorithms and number facts a few at a time at frequent intervals until they can be performed automatically. Timed tests are not recommended because they can put too much pressure on students and can cause them to become fearful and develop negative attitudes toward mathematics learning.</td>
<td>• allows for the development of efficient techniques for more complex procedures</td>
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**PROJECTS:** A strategy in which students work together to prepare and deliver a presentation or produce a product over a period of time.

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| Students choose a topic for a project. As the students research the topic, they link the topic with mathematical concepts. The project can be in the form of a term paper, a physical model, a video, a debate, or a mathematically relevant art, music, or athletic performance. | - builds on conceptual understanding  
- provides opportunity to engage in meaningful activities  
- develops problem-solving skills  
- allows for mathematical investigation of individual interests  
- encourages creativity  
- allows for integration with other subject areas |

**BRAINSTORMING:** A strategy for eliciting ideas from a group.

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| Students contribute ideas related to a topic. All contributions are accepted without initial comment. After the list of ideas is finalized, students categorize, prioritize, and defend selections. | - reveals background information and knowledge of a topic  
- discloses misconceptions  
- helps students relate existing knowledge to content  
- strengthens listening skills  
- stimulates creative thinking |
FIELD EXPERIENCE: A planned learning experience for students to observe, study, and participate in a setting off the school grounds, using the community as a laboratory.

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| Teachers and students plan and structure the experience before the visit and engage in follow-up activities after the trip. | • develops organizational and planning skills  
• develops observational skills  
• gives students an authentic educational experience |

LEARNING LOG: A strategy to develop structured writing.

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| During different stages of the learning process, students respond in written form under three columns: “What I Think”  
“What I Learned”  
“How My Thinking Has Changed” | • bridges the gap between prior knowledge and new content  
• provides a structure for translating concepts into written form |

GRAPHIC ORGANIZERS: A strategy in which teachers and students transfer abstract concepts and processes into visual representations.

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<th>HOW DO YOU USE IT?</th>
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| The teacher provides a specific format for learning, recalling, and organizing. | • helps students visualize abstract concepts  
• helps learners organize ideas  
• provides a visual format for study |
Graphic Organizer Strategies

**CONSEQUENCE DIAGRAM/DECISION TREES**

**WHAT IS IT?** A graphic organizer strategy in which students use diagrams or decision trees to illustrate real or possible outcomes of different actions.

**HOW DO YOU USE IT?**
Students visually depict outcomes for a given problem by charting various decisions and their possible consequences.

**WHAT ARE THE BENEFITS?**
- helps in transferring learning to application
- aids in predicting with accuracy
- develops the ability to identify the causes and effects of decisions

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Graphic Organizer Strategies (continued)

**Flowchart**

**What is it?** A graphic organizer strategy used to depict a sequence of events, actions, roles, or decisions.

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<th>How do you use it?</th>
<th>What are the benefits?</th>
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| Students structure a sequential flow of events, actions, roles, or decisions graphically on paper. | • fosters logical and sequential thinking  
• focuses on connections  
• develops the ability to identify details and specific points  
• develops organizational skills  
• aids in planning  
• provides an outline for writing |
Graphic Organizer Strategies (continued)

**VENN DIAGRAM**

**WHAT IS IT?** A graphic organizer strategy, derived from mathematics, for creating a visual analysis of information representing the similarities and differences among, for example, concepts, objects, and events.

**HOW DO YOU USE IT?**
Using two overlapping circles, students list unique characteristics of two items or concepts (one in the left part of circle and one in the right); in the middle they list shared characteristics. More than two circles can be used for a more complex process.

**WHAT ARE THE BENEFITS?**
- helps students organize knowledge and ideas
- helps students compare and contrast
- develops the ability to draw conclusions and synthesize
- stimulates higher cognitive thinking skills
**THE LEARNING CYCLE:** A sequence of lessons designed to have students engage in exploratory investigations, construct meaning out of their findings, propose tentative explanations and solutions, and relate concepts to their own lives.

**HOW DO YOU USE IT?**
The teacher engages the learners with an event or question to draw their interest, evoke what they know, and connect that with new ideas. The students explore the concept, behavior, or skill with hands-on experience. They explain the concept, behavior, or skill and define the terms, then use the terms to explain their exploration. Through discussion, the students expand the concept or behavior by applying it to other situations.

**WHAT ARE THE BENEFITS?**
- encourages students to construct their own understanding of concepts
- provides hands-on experiences to explore concepts, behaviors, and skills
- develops the ability to share ideas, thoughts, and feelings

**PROBLEM SOLVING:** A learning strategy in which students apply knowledge to solve problems. Problem solving should be a primary goal of mathematics instruction and an integral part of all mathematical activity.

**HOW DO YOU USE IT?**
The students confront a problem that might be somewhat vague, and use mathematical reasoning to formulate a mathematical version of the problem. The students represent the problem in more than one way. The students solve the problem using appropriate tools, then verify and interpret results with respect to the original situation. Finally, the students formulate additional questions or generalize solutions and strategies to new situations.

**WHAT ARE THE BENEFITS?**
- builds mathematical confidence
- allows students to discover relationships that may be completely new to them
- adapts easily for all grade levels and special-needs students
- develops the ability to construct new ideas and concepts from previously learned information, skills, and strategies
- develops higher level thinking skills
- allows students to experience the usefulness of mathematics
PREDICT, OBSERVE, EXPLAIN: A strategy in which the teacher shows the class a situation and asks students to predict what will happen when a change is made.

**HOW DO YOU USE IT?**
The teacher shows students a situation and asks them to predict what will happen when some change is made. Students observe what happens when the change is made. The class then discusses the differences between their predictions and the results.

**WHAT ARE THE BENEFITS?**
- encourages higher level thinking
- develops the ability to draw conclusions and synthesize

MODELS: A simplified representation of a concept. It may be concrete, such as Platonic solids or abstract like an algebraic equation or a geometric relationship.

**HOW DO YOU USE IT?**
Students create a concrete product that represents an abstract idea or a simplified representation of an abstract idea.

**WHAT ARE THE BENEFITS?**
- facilitates understanding of conceptual ideas

REFLECTIVE THINKING: A strategy in which students reflect on what was learned after a lesson is finished, usually by writing about what was learned.

**HOW DO YOU USE IT?**
Two possible approaches to reflective thinking are (1) students can write in a journal the concept learned, comments on the learning process, questions or unclear areas, and interest in further exploration, all in the students’ own words; (2) students can fill out a questionnaire addressing such questions as Why did you study this? Can you relate it to real life?

**WHAT ARE THE BENEFITS?**
- helps students assimilate what they have learned
- helps students connect concepts to make ideas more meaningful
Infusing a Multicultural Perspective

Florida students appreciate their own culture and the culture of others, understand the concerns and perspectives of members of other ethnic groups, reject the stereotyping of themselves and others, and seek out and utilize the views of persons from diverse ethnic, social, and educational backgrounds.

Florida's System of School Improvement and Accountability, Goal 3, Standard 10

Ethnic and cultural diversity enrich the American society and provide a basis for societal cohesiveness and survival. An effective program of multicultural education integrates a sensitive and thorough study of ethnic and cultural content into the curriculum. A carefully designed and continuous curriculum (preschool through 12th grade) can create the multicultural literacy so necessary for a healthy nation. Each cultural group has its own set of values and perspectives. Many of these values are shared with other cultures and form the basis of American national unity. Each cultural group has also made its own unique contribution to the American society and to the world. Because it is essential that all members of our society develop an understanding of the values and perspectives of racial, ethnic, and cultural groups, schools are restructuring their curricula to infuse multicultural perspectives into everyday instruction.

The presence of students with different cultural and family backgrounds, interests, and values in the same class encourages all students to develop a multicultural perspective. Learning settings that respect diversity encourage social competence and moral development. Students learn what they live. They learn to respect individual differences by understanding how others think and feel. Activities that promote empathy, understanding, and respect for differing points of view promote a multicultural perspective without negating one’s own point of view. Students learn to view concepts, issues, events, and themes from the perspective of diverse ethnic and cultural groups. Because the classroom is a model community, students gain the experience of living as responsible citizens in a diverse, democratic society.

Each student brings a wealth of culture that can be recognized, appreciated, and included as part of the instructional content. Teachers can focus on fostering understanding, appreciation, and respect for people of other cultural, language, socioeconomic, religious, or ethnic backgrounds, using the strengths and
backgrounds of their own students to enhance the school experience for all. Teachers can design learning activities that prepare students to communicate and work with others, achieving common goals in a culturally diverse environment. Schools can restructure their curricula to ensure that all students, regardless of background or ethnicity, will achieve high academic standards and be able to function successfully in the workplace. The final goal will be for students to have the cultural knowledge, positive attitudes, and motivation that will allow them to participate in a global community in which every person is respected, appreciated, and honored.

**Snapshot of an Effective Mathematics Classroom**

The following vignette is offered as an example of integrated, real-world educational experiences that teachers might create for students, using a variety of instructional strategies.

Mr. Anderson’s algebra students are learning how to write the equation of a line when given the coordinates of two points on the line. Because many students in past years found this topic difficult, Mr. Anderson has taken a suggestion from another mathematics teacher and is connecting the lesson to an activity involving statistics.

To increase interest in the activity, he begins by asking students to guess which is longer—their foot or the inside of their arm from the wrist to the elbow. The students measure and record the lengths to the nearest centimeter and record the data on the chalkboard in tabular form. Mr. Anderson then asks the students to graph a scatter plot of the data with arm length on the x-axis and foot length on the y-axis. Debbie and Eddie enter the data into a calculator which displays the data and the scatter plot on the overhead screen. The other students compare their graphs to the graph on the screen. Liz notes that points on the table appear only once on the screen even when recorded more than once.

Mr. Anderson asks the students whether they could make a prediction about the length of someone’s foot given his or her forearm length. After a discussion, the students agree that, in general, people with longer forearms have longer feet. However, the students find it difficult to make a numerical estimate from the collected data. Mr. Anderson uses his pen to draw a line through two points on the graph. The students notice that most of the plotted points fall on or near the line. Mr. Anderson helps the students find the equation of the line through the selected points. He then asks the students to pick two points from their plot and use the same procedure to write an equation for their line. After the students compare their equations, Mr. Anderson uses a calculator to generate the equation of the best-fitting line for the entered data. The
students then use the generated equation to predict the foot length of a person with a 34-
centimeter forearm length.

Later that day, Mr. Anderson reviews the results of the lesson process with another mathematics
teacher. He concludes that, although lessons such as this one take more time to complete than lessons
used in the past, the students demonstrated a greater understanding of the concept this year.

Mr. Anderson’s instructional approach represents a considerable change from typical
mathematics classes of the past. The teacher uses a tangible example to help students
achieve a higher level of understanding of difficult subject matter. Using a
cooperative learning strategy enables students to share their knowledge and work
together to more quickly pinpoint problem areas in their study.

Teaching Diverse Students

Schools must accommodate a diversity of student
abilities, disabilities, interests, cultural backgrounds, and
other factors that affect student performance in school. It
is important for all educators to be aware of the
characteristics of their students and vary their teaching
strategies to meet students’ individual needs. Many
instructional strategies that have been developed and
used by teachers for interacting with students with
special needs have proven effective for other students as
well.

Increasing ethnic and cultural diversity promises to
continue enriching life in the United States. This has
important implications for education. As diversity in
the school population grows, it becomes more and
more evident that all students, regardless of their race,
ethnicity, culture, and class, must acquire the
knowledge and competencies necessary for functioning
effectively with one another. All students must develop the knowledge and
competencies necessary to participate successfully in their communities, in the
workplace, and in society.
Adapting Instruction for the Diverse Needs of Learners

Given the focus on creating learning-centered classrooms, the unique characteristics of individual learners must guide curriculum planning and affect both the learning environment and the teacher’s role in facilitating the learning process. As curricula and learning environments are redesigned, and as teachers plan and teach, it is important to keep in mind that learners

- come to the educational setting with unique knowledge, experiences, and explanations about the world;
- come from many cultures and backgrounds;
- have diverse needs and values;
- actively participate in learning;
- have a variety of interests; and
- have a variety of opinions and ideas about school, mathematics, and the world.

Creating an effective learning environment that can address these diverse needs, backgrounds, and learning styles starts with understanding those needs.

Adapting Instruction for Developmental Differences

Children learn best when material is appropriate to their developmental level and challenges their intellectual, emotional, physical, and social development. Children grow through a series of definable, though not rigid, stages. Schools should demonstrate awareness and understanding of the developmental differences among all, including those children with special emotional, physical, or intellectual challenges as well as those with special abilities. Exploring the developmental differences of children in-depth is beyond the scope of this framework. Much research is available in this broad area.

Adapting Instruction for the Individual Learning Process

Children naturally develop unique capabilities and talents. They acquire preferences for how they learn and the pace at which they learn. There are many forms of intelligence and many ways by which people know, understand, and learn about the world.
Seven types of intelligences have been identified by Howard Gardner (1985):

- verbal/linguistic,
- logical/mathematical,
- visual/spatial,
- body/kinesthetic,
- musical/rhythmic,
- interpersonal (dealing with other people), and
- intrapersonal (knowing oneself).

Each student has a dominant learning style that consists of a unique combination of these intelligences. It is important for teachers to understand the learning styles of their students so that they can structure their teaching in a way that incorporates these seven ways of knowing. The mathematics program that matches teaching to learning styles allows students to process material more efficiently, thereby reaching all students and providing the opportunity for deeper and more thorough learning.

There are many other strategies for adapting instruction and the learning environment for students with different needs. One strategy might be to challenge students with, open-ended problems to which they can respond on a variety of levels. Teachers can enhance learning by encouraging students to explore on their own and frequently reinforcing their discoveries. Some students may need additional opportunities to practice previously mastered information. The mathematically limited or talented student may require different manipulatives than other students in their respective grades. For instance, a mathematically limited high school student may need to use base ten blocks, whereas a mathematically talented elementary student may be ready to use algebra tiles in the upper elementary grades. Situational problems should be adapted to relate to the developmental age of the exceptional student so that the context of the problem is relative to the student’s life. Instruction might take place in the form of individual activities, group activities, games, class discussions, or projects involving multiple skills. It may also be advantageous to vary class grouping to accommodate different tasks or learning styles.

Adapting instruction for the individual needs of students does not mean lowering expectations or having different academic criteria. The teacher’s high expectations for
academic success play an influential role in the way other students accept a student who has unique needs. This, in turn, can have an impact on a child’s self-image, affecting his or her eagerness and ability to learn.

**Accommodating Students With Disabilities**

Teachers who believe that all students can learn create a supportive learning environment for students with disabilities. In addition, modifications in assignments, courses, instructional methods, instructional materials and resources, and assessment methods can help enhance the learning experience for these students. Course modifications may be made to basic or vocational education courses in the regular classroom or in the exceptional student education classroom; these modifications are described in the State Board of Education Rule 6A-6.0312, FAC. Educators may modify a course by increasing or decreasing instructional time, that is, adjusting the time allotted for completing an assignment or a course or adjusting the length of class assignments. The format of the instruction can also be adapted or changed. This might include the use of hands-on materials, audio-visual media, instructional technology (including computers), and the use of specially designed materials such as the *Parallel Alternative Strategies for Students* (1992-1995), developed for Florida schools.

Quite often modifications that are effective for students with disabilities work well for other students in the class. Specially designed teaching strategies can be easily integrated into the classroom to enhance the content being presented, to assist with assignments, and to organize the content being learned. Testing modifications, such as flexible scheduling, recorded answers, use of mechanical aids, or revised formatting, may be helpful for all students.

Accommodating the needs of students with disabilities may include many other modifications. For example, there are students who need special communication systems in order to participate in classes. Students with hearing impairments may need the assistance of an interpreter or note-taker, or both. Computerized devices can help students with disabilities perform written and oral communication. Students with visual disabilities may require access to Braille and other adaptive technology.

When the needs of learners with disabilities are accommodated by modifying instructional methods, assessment methods, and the physical environment and by
providing a supportive environment such students are able to excel. They can
develop a greater capacity to take an active role in the learning process and focus on
their strengths, which helps them achieve a higher level of knowledge, skills, and
competencies in mathematics.

**Accommodating Limited English Proficient (LEP) Students**

Limited English Proficient (LEP) students are similar in most ways to students
whose heritage language is English: They learn at different rates, have various
interests and characteristics and different personalities, and bring vast differences in
background knowledge and experiences to the learning situation. All are unique.
However, language and culture add other dimensions to their uniqueness.

Problems may surface because these learners may use another language at home as
they are learning English at school. Thus there may be a psychological “pull”
between two worlds; these students often feel that their native language is “wrong.”
Because self-concept is influenced by the attitudes of others, negative attitudes from
family, friends, and school personnel may result in LEP students feeling isolated and
overwhelmed with the new environment, new sounds, and the new culture. Many
cultural references, idiomatic expressions, and multiple meanings of words that are
known to most literate English-speaking students may be foreign to LEP students.
An example might be the sign, “Fine for Loitering.” If the LEP student has learned
the meaning for “fine” as “it is all right to do something,” the sign would convey an
entirely different meaning than the idea of having to pay money for loitering. All of
these concerns may cause barriers to learning.

From the perspective of the teacher, teaching a multilingual class requires more time
and more effort because all students may not have similar background knowledge.
Teachers must be flexible, willing to learn and grow, be able to adapt and accept LEP
students, and value others’ languages and cultures. Many cultures have an entirely
different view of education, including the role of the teacher and the student, the
environment for learning, and materials used.

The following discussion of characteristics or behaviors educators may see in LEP
students is not meant to be a complete list or indicate that LEP students are progressing
in language development in the same way and at the same rate. Each student is unique,
and educators will need to consider the needs of each student individually.
As LEP students begin to learn English, they may

- remain silent; this should be accepted as a stage of language learning;

- depend on body language, gestures, or paralanguage (words or phrases such as “huh?,” “unh-unh,” and “uh-oh” usually accompanied by a facial expression and/or a gesture);

  The teacher’s consistency in structure, use of gestures, paralanguage, and body language is paramount.

- be actively listening as they silently translate;

  It is essential to remember that these students are not deaf and to wait for students to take the time they need to understand and formulate what they have to say.

- misinterpret body language or gestures;

  For example, a teacher’s motioning for a student to move toward her or him by using the forefinger may be viewed as a demeaning gesture in certain cultures.

- have limited school experience; and

  Some LEP students enter our school systems without much prior experience in school due to a number of factors in their native country.

- exhibit extremes of behavior: frustration, nervousness, fear, and self-consciousness.

As LEP students progress to an intermediate level in their English language skills, they may

- make unsystematic and random language errors that may lead to misunderstanding;

  Teachers should correct errors within the area of instruction rather than attempting to correct all errors. The latter leads to further frustration and an interruption in the thinking process of communication.
• exhibit social language skills in English that exceed language abilities necessary for academic success;

  Some young people quickly learn conversational English and mimic the actions of their peers, yet may have difficulty reading and writing appropriately. Conversely, some students are able to read and write in English, yet may have difficulty speaking.

• exhibit limited but continuing progress in vocabulary, control of sentence structure, ability to read with comprehension, and the ability to express ideas;

  It is important for teachers to continually provide opportunities for expansion of vocabulary and for use of vocabulary that has different meaning in specific contexts.

• generate language to ask and answer questions without being able to expand or explain; and

  Teachers should provide opportunities for LEP students to learn how to ask and answer questions that do not have a “yes” or “no” answer.

• require an extended period of time to translate information.

As LEP students move into the advanced level of language development and learning, they can begin to apply reading and writing skills to acquire information in academic areas and in real-life situations. These students may

• frequently choose to use more than one language to communicate;

  Teachers should learn to rephrase what the student has said in a correct model and focus on the use of English.

• exhibit oral fluency but still lack higher level, content-specific language and writing skills; and

• make inaccurate inferences from cultural, linguistic, and intellectual experiences.
Teaching Strategies

To support teachers of all subject areas in choosing effective strategies to use in working with Limited English Proficient students in their classes, the following suggestions are provided. It is important to remember that strategies may be introduced, extended, and expanded at all levels according to the interests and abilities of the learners.

At the beginning level, teachers may

- provide opportunities for students to hear and practice language in context with others;
  
  Remember that students need to listen to other students, other teachers, and people in the community to practice the sorting out of inflection, stress, intonation, and accent.

- provide a learning buddy or mentor;
  
  Peer support builds much needed friendships and understanding beyond academic areas.

- involve parents and community members; cultural exchange builds understanding;

- categorize words and ideas, which provides “hooks” for learning;

- use visual aids; label classroom items; match words with pictures, items, colors, and symbols;
  
  This helps students become familiar with physical areas of the school, for example, restrooms, the library, and the gym.

- provide opportunities for students to learn and respond to the usual classroom directions, for example, “raise your hand,” or “put your name in the upper-right-hand corner”; and

- use repetition and consistency in instructions and gestures.
At the **intermediate** level, teachers may

- set reachable goals and expect students to be accountable;
  
  *Teachers should demonstrate the correct model and expectations in the initial stages of an assignment or project.*

- encourage students to ask questions to clarify their understanding;
  
  *Making mistakes is seen as a step in the learning process, not something to be avoided.*

- obtain background information about language and culture to avoid embarrassing situations;

- speak clearly and at a normal pace with normal stress and intonation;

- check for understanding, as early clarification paves the way for success;

- present key words and ideas orally, on the chalkboard, and with the use of visual aids, before introducing new concepts; and

- use diaries, journals, or picture collages.
  
  *As learners have opportunities to express themselves in various ways, anxiety lessens.*

At the **advanced** level, teachers may

- provide examples when making assignments for book reports, class logs, lab reports, and research assignments; a visual goal helps with understanding;

- use cooperative learning groups; and

  *Collaboration within the mathematics class is a particularly useful instructional approach with students. For example, written and oral language develop, build, and “fine tune” from trial and error in collaborating groups. As students listen and participate, they learn to use gestures, tone, stress, and inflection to develop the “whole” of language, no matter what the content might be.*
• ask students to explain what they have heard or read and where they have seen words, phrases, or situations; this provides opportunities for expanding ideas and oral expression.

Generally, and across all subject areas, teaching LEP students requires

• knowledge of language development and language acquisition;

• the ability to adapt content to students’ needs and levels of learning;

• a willingness to learn about cultural differences and similarities;

• flexibility and sensitivity;

• a philosophy that learning takes place in every situation and in every environment;

• a belief that everyone learns from mistakes and from one another; and

• an encouraging, nurturing attitude.

Understanding and being sensitive to the needs of students who are learning English as a second language is important. Using effective strategies to support them as they learn mathematics will help ensure an environment that will provide successful experiences for LEP students.

**Accommodating At-Risk Students**

Students at risk of leaving school before graduation are a special challenge to the classroom teacher. Poor academic performance, as measured by being overage for a particular grade, in conjunction with grade retention and traditional and alternative assessments, has been cited as an accurate indicator of which students may drop out of school. Students who have difficulty meeting the required academic performance levels and who fall behind their peers often see little possibility of catching up; they may be at a high risk of not graduating.
Teachers can raise the level of student motivation by consistently modeling interest in the subject, tasks, and class assignments. They can also create classroom activities in which at-risk students are more likely to be successful and are able to tap into their own intrinsic level of motivation.

**Teaching Strategies**

Some strategies that have been effective in targeting at-risk students are the following:

- offering limited choices when it comes to alternatives for homework or long assignments;

- using active learning situations such as games, projects, group work, discussions, experiments, board work, creative seat work, and simulations (for example, mock elections, role playing, trials, and plays);

- providing concrete rather than abstract instruction, for example, physical objects, pictures, maps, diagrams, and colors as well as stories and anecdotes, because loading instruction with many examples makes the lesson come to life;

- using puzzles, brain teasers, and games to help students learn facts and figures;

- using short tasks and assignments, which provide more opportunity for completion, giving at-risk students a sense of accomplishment;

- having students compare their current efforts to their previous work rather than to the work of other students;

- avoiding class announcements of poor performance; avoiding posting or calling out grades;

- avoiding situations in which individuals compete openly in class; using, instead, group competitions in which teams are carefully designed so that the at-risk student is likely to meet success;
• helping students to concentrate on the task and its completion rather than on the consequences of failure;

• helping students evaluate situations in which they have been successful; helping students analyze unsuccessful situations and determine why they were unsuccessful; helping students focus on the path to success;

• teaching test-taking skills and avoiding timed tests;

• giving pretests so that students can make positive posttest comparisons, thus treating tests as opportunities for assessing learning rather than measuring ability;

• creating pretesting structures, for example, by providing study guides and outlines and teaching note-taking and outlining skills; and

• providing immediate feedback on student work by circulating around the classroom and monitoring students’ efforts on the spot, and promptly returning homework, assignments, and exams.

At-risk students, faced with a problem they have difficulty solving, often give up and simply go on to the next problem, or worse yet, do not even try to solve the problem and end up selecting answers randomly. The ability to persist can be taught. To encourage at-risk students to persist, teachers might

• carefully monitor students at work, coaxing them to continue working and to keep at it;

• help students set objectives and goals that bring immediate results;

• help students see that each new, small success brings them closer to their goals and makes them stronger;

• use contract learning, in which students have limited choices that move them step by step toward completion of course objectives;
- offer make-up exams, credit for effort, extra credit options, and extra practice opportunities;

- offer opportunities to rewrite or correct until revisions are completed; and

- help students retrace their work to find errors, analyze problems, and reread portions they have skipped in order to answer the questions.

The Dropout Prevention Act of 1986, Section 230.2316, Florida Statutes, was enacted to authorize and encourage school boards to establish Dropout Prevention Programs. These programs are designed to meet the needs of students who are not effectively served by traditional programs in the public school system. This includes students who are unmotivated, unsuccessful, truant, pregnant and/or parenting, substance abusers, and disruptive, as well as those who are in shelters.

Strategies used in these programs that have been found to be effective could prove successful in a more traditional setting. These include

- instructional strategies and tools such as cooperative learning, computer-assisted instruction, authentic/alternative assessment, critical thinking, and graphic organizers;

- competency-based curriculum which allows students to work at their own pace;

- flexible scheduling or use of time;

  Students “declare” a schedule and attend, even though it may be beyond the traditional school day. Competency-based curriculum delivered through computer-assisted instruction is well suited to this strategy.

- career awareness and on-the-job training for employability skills;

- experiential learning and hands-on activities;

- mentoring and nurturing;
• course modifications;
  Course modifications allow at-risk students to compress or extend the period of
time it takes to master material in a given course, to respond to a variety of
assessments to demonstrate mastery, and/or to be offered interdisciplinary or
intradisciplinary units of instruction through the integration of more than one
course description. This gives the overage-for-grade students an opportunity to
catch up with their own grade peers.

• summer bridge programs;
  Summer bridge programs allow overage-for-grade students to catch up with
their own grade peers by attending a rigorous summer session and then being
promoted to the next grade level.

• collaborative teaching that combines two classes;
  In one model of collaborative teaching, the dropout prevention teacher furnishes
expertise in course content, while the specific learning disability teacher offers
expertise in course modification.

• thematic units in which teachers identify common themes and realign
student performance standards to reflect the theme;
  In some models, teachers work together to identify aspects of their discipline
that have commonalities; in other models, teachers work separately without
any attempt to connect with other subject areas.

• peer counseling and student conflict mediation;
  One model pairs at-risk ninth graders with twelfth graders who are selected
according to leadership skills and their potential to serve as role models, and
who are trained in peer counseling strategies including listening, questioning,
paraphrasing, and feedback. These older students also provide academic
 tutoring and use a variety of peer counseling strategies designed to help the
ninth grader become successful in an academic curriculum that addresses
social, individual, school, and family concerns; topics could include drug and
alcohol abuse, family relations, academic motivation, and coping with stress.
• student support and assistance components, which serve students who are eligible for dropout prevention programs and who are in need of academic or behavioral support;

Students are served in traditional classes through a flexible schedule of auxiliary services, including supplemental materials or alternative strategies to assist with course modification, behavior management, or alternative assessment. Instructional aides or case managers can also be used to support teachers, students, and parents.

• GED/HSCT Exit Option; and

This program allows currently enrolled, dropout-prevention students to earn a standard high school diploma by enrolling in courses for credit that lead to a standard high school diploma and work to master the individual course student performance standards. To enter the program, these students must be behind the class with whom they entered kindergarten and demonstrate probability for success on the GED through documentation of a high score on a standardized test; to complete the program, students must complete required courses and pass the HSCT and the GED tests.

• coordination with other agencies, such as social service, law enforcement, prosecutorial, and juvenile justice agencies as well as community-based organizations.

**Putting These Ideas to Work**

Current educational philosophy recommends that educators focus on developing a learning-centered curriculum, which includes a number of key ideas:

• The teacher is a facilitator (a “guide on the side” versus a “the sage on the stage”).

• The student is a discoverer of knowledge within his or her learning community. This involves students listening to others and learning to filter information and draw conclusions, versus simply taking in a body of knowledge imparted by the teacher.

• The community is a rich resource.
• Real-world learning experiences help students apply knowledge and skills; this helps prepare them for daily living and future employment.

Using the curriculum frameworks as a guideline, local educators will make the final choices regarding how to teach the essentials. These choices will include the themes and topics by which to teach academic standards, the day-to-day content of instruction, the types of materials and resources used, and the teaching strategies that are appropriate for the individual needs of the students and for the teacher’s own strengths. The result of a thoughtfully designed curriculum is students who have the ability to achieve high academic standards and who can be better prepared to live as responsible, effective, and productive citizens within a global society.
Key Chapter Points

Instruction that prepares students for the 21st century should focus on

• high academic standards with expectations of high achievement for every student;

• a learning-centered curriculum with the teacher as a facilitator of learning;

• learning based on constructing meaningful concepts from facts;

• learning mathematics in its real-world contexts;

• making connections within mathematics and with other content areas;

• relating mathematics to the students’ world;

• active, hands-on learning in the classroom;

• more student responsibility and choice;

• students inquiring, problem solving, conjecturing, inventing, producing, and finding answers;

• students working and learning cooperatively;

• accommodating individual student needs, whether cultural, developmental, or cognitive;

• infusing a multicultural perspective;

• expanding resources to include local and global communities;

• using technology to support instruction; and

• relating classroom learning to the skills students will need to function successfully in their communities, in the workplace, and in society.
Why should teachers try to connect mathematics to other subject areas? There are at least three compelling reasons for doing so. First, life does not occur in neat, subject-matter packets. A single incident, such as a hurricane, affects a region in many ways. It destroys homes, cultural resources, and businesses; damages the environment; upsets the economy; interrupts school and school programs; tests government emergency response policies; and demands immediate solutions to problems that will have a long-term aesthetic and economic impact upon the quality of life in a community. To address these complex issues, citizens must integrate and use knowledge and skills from a variety of subject areas. Second, making connections among subject areas creates a greater sense of meaning for students; for example, a process they learn in mathematics helps them better understand social studies. Finally, today’s teachers face the challenge of an ever-expanding curriculum. Although the expansion of the curriculum results in part from important mandates from the state level, most of it results from the simple fact that information in the modern world is expanding at a phenomenal rate. This expansion of information underscores the importance of stressing connections among subject areas.
Curricular Connections and the Transfer of Learning

Connecting important concepts from different disciplines has a number of beneficial effects. One of the most important effects is that it facilitates the transfer of learning. A disappointing fact about education in America is that students frequently demonstrate that they understand something in one setting, but fail to understand that same concept in another setting. Educators refer to this occurrence as a lack of transfer of learning. For example, a student might show that he or she understands how to use exponents when asked to describe them in a mathematics class, but fails to see how that same concept applies to population growth in history class. By forging connections among subject areas in the classroom, students have a better chance of recognizing that what they learn in school has applications beyond the classroom. This transfer of learning is illustrated in the following example:

Carla's sixth-grade mathematics class is in the middle of a unit on measurement. Carla's teacher explains that as the dimensions of a figure, such as length, width, height, and radius change, other measurements such as perimeter, surface area and volume also change. Carla works in class and at home on problems that challenge her to use this new knowledge.

This weekend Carla has a babysitting job at a neighbor's house. The neighbor gives Carla a twenty-dollar bill and asks her to order pizza for herself and the children. When she places the order, Carla learns that she can order either two six-inch pizzas for $5.50 or a 12-inch pizza for $9.95. She finds that after working during the week on similar problems in mathematics class, she is able to quickly calculate which offer provides more pizza for the money. When the children's parents return that evening, Carla relates her experience of ordering the pizza. The parents are delighted to learn that Carla saved them money and give her a bonus in addition to her babysitting pay.

Curricular connections also encourage teachers to work in a collaborative mode. Most teachers have heard the expression, “Teaching is one of the most isolated professions in the world.” Fortunately, it doesn’t have to be. A mathematics teacher who decides to use content from the arts creates a reason to interact with the arts teacher. The interaction among teachers from different content areas can take many forms, depending on the model that is being used for making curricular connections.
Models for Curricular Connections in Instruction

Several strategies will be overviewed in this chapter; curriculum developers and teachers may want to explore these strategies in greater depth. Four effective models of curricular connections are infusion, parallel instruction, multidisciplinary instruction, and transdisciplinary instruction. After further exploration of these models, individual school staff must determine whether any or all of these models will work in their setting.

Infusion

In infusion, a teacher in a given subject area integrates another subject area into his or her instruction.

Sixth-grade students are studying the concept of percentages in Mrs. Whalen’s mathematics class. To illustrate the concept in pictorial form, Mrs. Whalen draws a pyramid on the board and sketches in the percentage of each food group recommended by the American Dietary Association for daily nutritional requirements. She invites the students to carefully record what they eat during and between each meal for the next three days. Working in groups, students calculate what percentage of their daily diet actually falls into each food group category and then compare their actual diet to the recommended daily food requirements. Mrs. Whalen asks her students to conduct a self-assessment of their diets and then write a paper that explains their findings and includes a plan for improvement.

Parallel Instruction

In parallel instruction, teachers from different subject areas focus on the same theme, concept, or problem. Each discipline is taught separately, but teachers must plan together to identify the common element and determine how the concept, theme, or problem will be addressed in each subject area. Homework and assignments commonly vary from subject area to subject area, but all reflect the common theme, project, problem, or concept being addressed.

High school teachers decide to explore the theme of balance as it applies to the content areas of science, social studies, language arts, health, the arts, and mathematics. Although each subject area deals with “balance” in different ways, teachers meet prior to the unit to collaborate on...
ways that the concept might be presented and to ensure that the concept will be reinforced in each subject area.

In science, students explore the importance of balance in nature and the environmental consequences that result when certain aspects of nature are not in balance. Specifically, students focus on clean air standards and methods of lowering levels of air particulates to maintain a healthy balance. Groups of students visit industrial sites in the area and measure levels of particulate matter in the air at those sites, noting prevailing winds and how these might affect air quality in their neighborhoods. Students present their findings to an EPA representative who had visited their class to speak about local efforts to keep the air clean.

Social studies teachers discuss the concept of checks and balances built into American government. Students also study how checks and balances are factors in creating the federal budget. The students contact the congressperson representing their area; she speaks to the social studies classes about her budgeting experiences as a member of the Ways and Means Committee.

In language arts, students are in the midst of studying persuasive techniques and debate. Topics for debates have "balance" as their focus; these include eliminating certain federal programs as a means of balancing the budget, hunting as a means of balancing predators and prey, the pros and cons of balancing school populations through enforced busing, and balancing environmentalists' concerns with those of pleasure boaters on environmentally delicate intracoastal waterways.

Health classes study the importance of balance in diet and exercise. Students keep track of their diets for a week and are surprised to note the differences between what they typically ate and the balanced diet recommended by the American Dietary Association. They also track whether their leisure time includes a balanced amount of exercise. A panel consisting of a physician, physical therapist, and dietitian from the local hospital speaks to health classes on food addictions and how imbalances in food-group intake can affect physical and emotional health.

The art teacher takes his students to the museum; students work in groups to identify and critique the use of balance and symmetry in a variety of artwork and sculpture they had seen during their field trip. Next, they design their own artwork to illustrate the concept of balance; these pieces are displayed throughout the building.
In mathematics, students investigate the concept of balance in a number of areas. One of these involves how balance is a critical factor that architects and engineers apply in the design of multilevel structures and bridges. Students also study how statisticians use measures of central tendency to display the balance and dispersion in sets of data. Students create drawings and graphs to demonstrate what they have learned; for example, one group gives a class presentation explaining the relationship between the size and depth of a foundation and the number of floors of a building.

**Multidisciplinary Instruction**

As with parallel instruction, within multidisciplinary instruction two or more subject areas address a common concept, theme, or problem. The subject areas are taught separately for the most part, but a common assignment or project links the various disciplines. Teachers must plan together to identify how the concept, theme, or problem will be addressed in each subject area, construct the common project, determine how the project will be divided among the subject areas, and determine how students will work together on the project.

Middle school students work together on a project developed by their foreign language, social studies, mathematics, and language arts teachers. In order to demonstrate an understanding of what they had learned in each of their classes, students are given the assignment to produce a special children’s information section featuring a Latin American locale for the local paper. The students decide to focus on Guatemala. While studying Spanish, students use sources written in Spanish to conduct research about the Guatemalan culture, gathering information on food, sports, games, music, art, festivals, and lifestyle. They also use Spanish-language tourism publications to discover how and why certain places of historical significance are important to Guatemalan citizens. While studying social studies, students research Guatemalan history, government, economy, and its role in Latin American and world affairs. In mathematics, students study maps of Guatemala to determine distances between cities and calculate time needed to travel between points of interest. In language arts, students study magical realism, a popular literary form in Latin American culture. They decide to include a book review of a novel written by a Guatemalan author in their information section. Working in groups,
students use the research they have compiled in Spanish, social studies, mathematics, and language arts to write articles which are submitted to the local paper for the children's section.

**Transdisciplinary Instruction**

Within transdisciplinary instruction, two or more subject areas address a common concept, theme, or problem; however, the subject areas are presented in an integrated fashion. Classes in the subject areas meet at a common time; teachers integrate planning and team-teach all lessons. Commonly, a major project is the focus of the unit.

A mathematics teacher and a language arts teacher at the high school decide to collaborate on a team-taught project to demonstrate how an understanding of mathematics can be an important tool in literary analysis. The teachers choose Edgar Allen Poe's "The Pit and the Pendulum" as the focus of the unit. The language arts class and mathematics class meet back-to-back during the school day; the teachers of these classes also share a common planning period, during which time they construct, modify, and evaluate the two-week unit. During the two-week unit, the mathematics and language arts classes meet as one team-taught block class. The teachers determine that the objectives of the unit will be to have students study the concepts of time, space, and mechanics in the story. Students will analyze the narrator's descriptions of his surroundings and the details of his imprisonment, then use mathematics to determine the credibility of his assessments. Next, students will conduct a literary analysis of the story and discuss the significance of their mathematical findings to the narrative constructions in the story.

To complete this assignment, students spend time in class analyzing the narrator's description of the dungeon. They use this information to calculate the dimensions of the dungeon, the location of the pit in the room, and the location of the pendulum. They also use the narrator's descriptions to determine the frequency of the pendulum's swing, the arc of its sweep, and the amount of time that elapses in the story. Students use their calculations to build a scaled model of the dungeon, incorporating all of the details provided by the narrator. From this model, students ascertain whether the pendulum would be operated manually or mechanically, whether the dungeon could really exist as the narrator described it, and whether the narrator's escape is feasible.

After constructing their model, students participate in a class discussion on the ways in which Poe used the details of the dungeon to construct the story. The teachers encourage the students to consider how their mathematical analysis of the dungeon affected their understanding of theme, mood, and setting in the story. At the end of the unit, students write reports in which they
discuss their thoughts on the narrator’s credibility in the story, using both their understanding of first-person narration and their mathematical findings to support their conclusions.

Planning an Interdisciplinary Unit

One of the most effective ways to plan a unit that fosters connections is to focus on creating projects that involve content from different subject areas. As we have discussed, projects are a central part of both multidisciplinary and transdisciplinary instruction. Below is a simple three-step process that can be used to develop projects that forge curricular connections.

Step #1: Select benchmarks from two or more subject areas that will be integrated into the project.

For example, assume that a mathematics teacher sets out to construct a project that incorporates a benchmark from social studies. She would first consult chapter 3 of this framework and select a benchmark. For example, she might select the following benchmark which can be found under the geometry and spatial sense standard: “The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed”:

Mathematics benchmark: The student understands the concepts of spatial relationships, symmetry, reflections, congruency, and similarity.

The teacher would then consult the framework for a second content area—the arts, for example. The teacher might pick the following benchmark which can be found under the visual arts standard entitled “The student understands and applies media, techniques, and processes”:

Visual Arts benchmark: The student knows the effects and functions of using various organizational elements and principles of design when creating works of art.

These two benchmarks—one from mathematics, one from the arts—would form the basis for the project. It is important to realize that all benchmarks must be selected with a great deal of attention to their relatedness. In other words, not all pairs of benchmarks make a good match. The two benchmarks chosen here correlate because the processes in the visual arts benchmark can be applied to the concepts described
in the mathematics benchmark. If a teacher tries to force a connection between benchmarks from different content areas, the resulting project will be artificial and will run the risk of confusing students or will not maintain the integrity of the chosen subject areas.

**Step #2: Identify an interesting question or questions that can be asked about the benchmarks that have been selected.**

One way to help students explore the relationship between benchmarks is to ask a question that will naturally integrate the benchmarks. The following is a list of useful questions to consider:

- What is the underlying pattern?
- How are these things similar and different?
- What groups can these things be put into? What rules or characteristics have been used to form groups?
- What conclusions can be formed about this information?
- What is the evidence for this position and how good is it?
- What specific rules are operating here? Based upon those rules, what must happen or what will probably happen?
- Are there errors in reasoning that have been made? Are there errors being performed in a process?
- Is there a hidden relationship here? What is the abstract pattern or theme that is at the heart of the relationship?
- Are there different perspectives on an issue that should be explored?
- Is there some new idea or new theory that should be described in detail?
- Is there something that happened in the past that should be studied?
- Is there a possible or hypothetical event that should be studied?
- Is there an obstacle that must be overcome?
- Is there a prediction that can be generated and then tested?
- Can this skill or process be used to accomplish something or better understand something?

**Note:** Adapted from Marzano, Pickering, & McTighe. (1993). *Assessing Student Outcomes.*
To illustrate how a question from this list can be used, consider the two benchmarks that have been selected:

**Mathematics benchmark:** The student understands the concepts of spatial relationships, symmetry, reflections, congruency, and similarity.

**Visual Arts benchmark:** The student knows the effects and functions of using various organizational elements and principles of design when creating works of art.

A question that seems to naturally address these benchmarks is, “Can this skill or process be used to accomplish something or better understand something?” In other words, how can students use organizational elements and principles of design to demonstrate an understanding of concepts associated with spatial relationships?

**Step #3: Identify a product or products that incorporate the benchmarks that have been selected.**

With the content benchmarks selected and an interesting question identified, the next step is to identify the product or products that best suit the project. It is useful to consider four types of products: (1) conclusions, (2) processes, (3) artifacts, and (4) affective responses. It is important to remember that some products may not be applicable to all subject areas.

**Conclusions** are generalizations that have been constructed as a natural consequence of studying some issue or topic. For example, in mathematics, students might collect data about a particular type of stock, graph the data, and use the graph to draw a conclusion about the stock’s performance over time. When students report their conclusions, they commonly are expected to provide evidence and support. This may be in the form of oral or written reports, videotapes, audiotapes, charts, and graphs.

**Processes** are sets of actions that are the natural consequence of solving a problem or accomplishing a goal. For example, in mathematics, students might be asked to use their knowledge of ratio and proportion to develop a process for measuring the distance between planets in our solar system. Processes are commonly demonstrated along with an explanation of how the process works and why it is effective. If the process cannot actually be demonstrated, it is sometimes simulated.
Artifacts are physical products that are natural consequences of solving a problem or accomplishing a goal. It is important to distinguish artifacts from products, which are demonstrations of conclusions. Examples of artifacts in the mathematics classroom include tessellations, two- and three-dimensional geometric designs, optical illusions, and fractals. Affective representations are illustrations of emotional responses that are a result of studying some issue. They take many forms including paintings, murals, sculptures, collages, sketches, and personal essays. Affective representations may be less applicable to mathematics classrooms.

Of these four types of products, an artifact seems to be the one best suited for the project using organizational elements and principles of design to better understand the concepts of spatial relationships, symmetry, reflections, congruency, and similarity. The project requires students to create a work of art that demonstrates a knowledge of mathematical principles. With the benchmarks selected, an interesting question identified, and a type of product selected, the teacher would then write the project as a set of directions to the students. Those directions might read as follows:

In art class, you are learning how various principles of design are used to create works of art. Those same elements can be applied to mathematics to help us visualize how shapes can be combined, subdivided, and changed for a variety of purposes. For our next class project, we will be creating tessellations. I want you to break into small groups to complete the assignment. Each group will have a piece of cardboard approximately 5' by 5' on which to work. This board will serve as your plane. Your tessellation must cover the plane completely, leaving no gaps and without overlaps. In your groups, you must work together to choose two different pattern blocks for your tessellation. You will be provided with measuring tools, colored paper, and scissors to create your design. You may want to try sketching out your design on paper before you begin to cut and paste. Your tessellations should illustrate the concepts of congruency, symmetry, reflections, and similarity. After you have finished your tessellations, each group will present its work to the class. During the presentation, you will discuss how you created your pattern, including the design principles at work. You will also point out the spatial relationships illustrated in the pattern. The visual arts teacher has agreed to let us display our work in the student gallery so that we can demonstrate for other students in the school how mathematical concepts can be applied to the arts, and arts concepts can be applied to mathematics.

As this example illustrates, creating a project that involves benchmarks from different subject areas is a complex process. However, it is worth the effort in terms of student motivation and learning.
Key Chapter Points

- There are four basic ways in which curricular connections can be forged: infusion, parallel instruction, multidisciplinary instruction, and transdisciplinary instruction.

- A three-step process can be used for constructing projects that forge curricular connections.

- Curricular connections make learning more meaningful for students.
Assessment of student academic achievement is a fundamental component of Florida’s school improvement and accountability initiative. Assessment provides essential information on the effectiveness of our reform efforts and on the level of student achievement of Florida’s academic standards. Assessment processes are varied and include the use of standardized tests as well as other formal and informal methods to build a web of useful information about student achievement.

Florida schools will be held accountable for student achievement through the collection and analysis of academic assessment information from the state, district, school, and classroom levels and the public reporting of results. One highly visible part of the education accountability program will be a statewide, externally mandated assessment system measuring student progress in reading, writing, and mathematics in a context of high-level thinking and problem solving. This state test will provide an external “spot check” on the first four standards of Goal 3. This system will be criterion referenced and will include performance-oriented items. It will be administered at three levels: elementary, middle, and high school.

A statewide assessment program, however, is not adequate by itself to provide all of the information on student knowledge and skills needed at the local level. This can only be provided through the proper use of classroom assessment procedures. The focus of this chapter is classroom assessment, one of the teacher’s most complex and most important responsibilities. This chapter presents overviews of various strategies
for classroom assessment. Curriculum and assessment developers and teachers should explore these strategies in greater depth through other more-detailed sources.

**Classroom assessment** refers to the tasks, activities, or procedures designed to obtain accurate information about student academic achievement. Assessment helps answer these questions: What do students know and what are they able to do? Are the teaching methods and strategies effective? What else can be done to help students learn?

Classroom assessment activities should be systematic, ongoing, and integrated into the process of instruction and learning. This dynamic relationship results in a continuous process of refining goals as the teacher works with the entire class and with individual students. In fact, the term assessment comes from the Latin *assidere*, which means “to sit beside.” This meaning creates a picture of the teacher and the student working together to continually improve the processes of teaching and learning. To assess also means to analyze critically and judge definitively. This meaning emphasizes the teacher’s responsibility to make judgments about students’ achievement based on careful consideration of obtained information.

Authenticity in classroom assessment activities is desired whenever possible. That is, assessment activities should not only examine simple recognition or recall of information, but should also determine the extent to which students have integrated and made sense of information, whether they can apply it to situations that require reasoning and creative thinking, and whether they can use their knowledge of mathematics to communicate their ideas. Using authentic (i.e., realistic) assessment activities will help reveal whether or not students have learned to do these things. The strategies presented in this chapter will encourage the linkage of curriculum, instruction, and assessment.
The Assessment Process

In recent years, our knowledge of how students learn has increased; for example, we have learned that students acquire knowledge and skills in widely diverse ways. Knowing this, however, only serves to increase the complexity of student assessment. Because all students do not learn in the same way and because increasing numbers of our students come to school from situations that seriously affect their prospects for success, innovative approaches to instruction and assessment are needed to meet their needs.

The process of assessment is not complete without the communication of results. Timely feedback from assessment is important to positively influence student performance and instruction. Comments about student progress may be formal or informal and should emphasize what students have done successfully and what they have achieved. The process should include opportunities for the student to comment on his or her own progress and for the student’s family to be involved in and informed about the assessments. Summary results of classroom assessments should be shared with other educators, citizens, and decision makers, where appropriate, and used by educators to improve instruction.

Different Types of Classroom Assessment

The unique nature of mathematics calls for using multiple forms of assessment to clearly evaluate each student’s progress as well as the impact of instructional strategies. The task of teachers and assessment specialists is to use the most effective and valid forms of assessment for the particular educational setting, for the type of knowledge, skill, or ability being assessed, and for the individual student. Developing a variety of assessment options allows the teacher to match the assessment to the student’s ability to demonstrate knowledge to verify that learning has taken place.

Even when a variety of options is available, modifications for specific students may also be necessary. Modifications that are made in the classroom for the instruction of special needs students often can be applied to assessment procedures. For example, it may be more effective to allow a student the opportunity to give an oral presentation rather than a written report.
When written tests are used to assess student performance, test administration can be modified in a variety of ways, including flexible scheduling and flexible settings. Students may perform better if not hampered by artificial time limits or disrupted by other students in the class. Using a revised format that may allow the student to listen to test questions rather than read them can also improve performance for students with reading disabilities. Recording answers or performances via audiotape or computer programs may help a student demonstrate competency under less stressful circumstances.

Assessment techniques overlap and blend together. Using several forms of assessment provides a broader and more comprehensive picture of the learning and teaching of mathematics. Educators are encouraged to select from among the many innovative assessment strategies available, a number of which are described below.

**Traditional Assessment**

Traditional assessment is a term often used to describe the means of gathering information on student learning through techniques such as multiple-choice, fill-in-the-blank, matching, or true/false questions, and essays. These approaches are particularly useful in assessing students' knowledge of information, concepts, and rules.

Because factual knowledge of information is one important aspect of mathematics, carefully designed multiple-choice, true/false, and matching questions can enable the teacher to quickly assess the building blocks of the mathematics curriculum. Examples of such skills include the following: Can the student recognize important terms, relationships, and symbols? Does the student recognize how knowledge is organized into patterns, how generalizations are formed from evidence, how events are understood in chronological order, how frames of reference inform decision making, and how predictions can be made from data?

Effective assessment evaluates knowledge of facts as well as their connection to a broader body of knowledge. Proficiency in mathematics depends on the ability to know and integrate facts into larger constructs. For example, students must know how to apply arithmetic facts in real-life situations in order to demonstrate a broad conceptual understanding.
Assessment Alternatives

There are many “alternatives” to traditional assessment that can be used to broaden the scope of the teacher’s classroom assessment activities. In some of these alternative assessment forms, students perform self-evaluations of their work. In others, teachers make informal observations about students’ knowledge, skills, and performance that relate to subject-area topics.

The following list of alternative assessment techniques is by no means exhaustive. New assessment techniques are continually being developed to measure students’ progress toward achieving new academic performance standards and benchmarks.

**Performance assessments** require the student to create a product or construct a response that demonstrates a skill or an understanding of a process or concept. Performance assessments are commonly presented to students as projects that are done over an extended period of time and require that students locate, gather, organize, interpret, and present information. Typically, the project or product of the assessment is rated by the teacher or team of teachers using clearly delineated criteria.

Mrs. Cuevas wants her students to understand how algebra, geometry, and computation can be used together to solve problems. She sends her students to the gym in groups of three and four to measure the basketball court. The students measure the floor layout, including all lines, curves, and basket positions. After returning to the classroom, students use algebraic strategies to determine the correct measurements for constructing scaled models of the court. The students build their models with the help of graph-paper diagrams and computer simulations. During this activity, Mrs. Cuevas observes each group to determine how the students use pictorial models and technology as problem-solving devices. She asks the students to explain the mathematical equations they used and their reasons for using them. Mrs. Cuevas is then able to assess each group’s model based on the rubric she had developed with the class before the activity began.

**Authentic assessments** are a form of performance assessment structured around a real-life problem or situation. Although traditional multiple-choice questions can describe real-life situations, the term “authentic assessment” usually is applied to performance assessments.
Students in Mrs. Martinez's class think that the price of pizza in the school cafeteria is too high. Mrs. Martinez decides to use this issue in a problem-solving activity in which her students will propose a fair pricing system for school-lunch pizzas. This activity allows students to apply their computational and analytical skills to an issue of direct relevance to them. Before the activity, Mrs. Martinez and her students establish criteria for the pricing proposals and develop a scoring rubric. Mrs. Martinez has her students research the prices of ingredients and estimate the cost of making each pizza, the cost of labor, and overhead expenses. She also asks them to incorporate the concepts of supply and demand that they are learning in social studies. Students come up with fair pricing proposals and present them to the class.

**Teacher observation** is a form of data collection in which the instructor observes students performing various activities without interrupting the students' work or thoughts. Teachers use checklists, rating scales, or notebooks to record their judgment about students' competence in specific standards or benchmarks.

Mrs. Landers wants to determine how well her students understand the trigonometric concepts explored in class recently. She observes Dana and Sudha working together on an assignment to use the computer to model the path of a piece of chewing gum on a tire of a moving car and develop an appropriate equation that could be used to graph the path of the gum on the tire. Mrs. Landers takes notes as she watches the girls complete the assignment. By listening to Sudha answer Dana's questions, Mrs. Landers is convinced that Sudha has a firm understanding of the trigonometric concepts used in the assignment. She also realizes that Dana is having some trouble understanding the differences and similarities between sine and cosine functions. Mrs. Landers meets with Dana to help clarify key concepts.

Interviews require students to respond verbally to specific oral questions. The instructor asks questions, interprets answers, and records results. This form of assessment also allows a teacher to discuss student answers, to probe for more complete responses, and to identify misconceptions so they can be corrected. Correction should be postponed until the interview is completed to encourage the free flow of ideas and to reduce student apprehension.

Mr. Volien asks Mark to explain the steps he used to solve a problem related to area and volume. After each step, Mr. Volien asks Mark questions to determine how Mark made his choices. Mr. Volien asks questions such as, What is this problem about? How would you solve this problem? How would you explain the process to a younger child? What do you predict will happen? Can you think of another way to solve the problem?
Conferencing involves a two-way dialogue between a teacher and students or among students for the purpose of evaluating progress on a specific standard or benchmark or on a project.

Ms. Rao has introduced her class to concepts associated with algebraic expressions. She gives an assignment in which the students translate verbal mathematical statements into algebraic expressions. When Seth completes the assignments, he meets with Ms. Rao to discuss his answers. Ms. Rao begins by asking Seth if he has any questions about the assignment. Seth isn't sure he understands the concept of variables. Ms. Rao encourages Seth to come up with real-world examples to help him visualize how variables can be used to solve problems. During the conference, Seth discusses and compares various problem-solving strategies with Ms. Rao. The conference allows Seth to articulate his questions and gives Ms. Rao the opportunity to discover how Seth approaches problem-solving situations and how she can most effectively guide his learning.

Self-assessment enables students to examine their own work and reflect upon their accomplishments, progress, and development. The teacher may supply the student with assessment criteria or assist students in developing their own. This form of assessment assists students in developing the critical thinking and evaluative skills that lead to effective problem solving and independent learning.

Mr. Lai wants his students to understand how graphs can function as tools to analyze data, so he gives his class the assignment of taking a simple survey of their friends' after-school activities. Working together as a class, the students write the questions for the survey. After collecting the results of the survey, each student creates a pie chart, a bar graph, and a line graph of his or her findings. After Camille completes the assignment, she evaluates her graphs according to the questions Mr. Lai asks the class. How do her graphs illustrate her findings? Does one of her graphs illustrate her findings more effectively than the others and why? What has she learned from making her graphs?

Peer assessment involves students evaluating each other's work using objective criteria. It requires students to reflect on the accomplishments of their classmates. By assessing others' work, students often see alternative reasoning patterns and develop an appreciation for diverse ways of approaching and solving problems.

Students in Mr. Colby's class are given an assignment in which they will explore the characteristics of triangles. Working in groups, the students use computer software with geometric manipulation capabilities and present their discoveries to the class. After the students
explain their conjectures to the class, including the steps they took to reach their conclusions. Mr. Colby asks the students to develop a rubric to assess and provide feedback on the representations and justifications of their conjectures.

**Portfolio assessment** is a purposeful collection of a student’s work that provides a long-term record of the student’s best efforts, progress, and achievement in a given area. Materials included may be decided on by the student, the teacher, or both. Depending on the intent, portfolios can serve as the basis for assessing individual student growth over time on given standards and benchmarks, or for assessing learning specific to the objectives addressed in a theme or unit. It is important to note that, although a portfolio can be used as an effective instructional tool, its use as an assessment tool demands a clear understanding of purpose, specification of the desired portfolio contents, and a definition of the methods of rating the individual components of the portfolio.

Students in Ms. Duchaine’s algebra class are creating portfolios of their work thus far in the semester. Ms. Duchaine encourages her students to select work that they feel reflects growth, change, or understanding over the semester. Benjamin meets with Ms. Duchaine in order to determine what to include in his portfolio. After much thought, Benjamin decides to focus on the progress he has made in understanding algebraic concepts. With each assignment he includes in his portfolio, Benjamin writes a paragraph about why each item was chosen and summarizes his growth in understanding over the semester. Ms. Duchaine writes a review of Benjamin’s portfolio in which she assesses Benjamin’s progress and praises his grasp of algebraic concepts.

**What could go into a mathematics portfolio?**

A portfolio should capture the richness, depth, and breadth of a student’s learning within the context of the instruction and the learning that takes place in the classroom. The focus of a students’ portfolios should be on student thinking, growth over time, mathematical connections, students’ views of themselves as mathematicians, and the problem-solving process. Elements of a portfolio can be stored in a variety of ways; for example, they can be photographed, scanned into a computer, or videotaped. The possible elements of a portfolio include selected students products such as the following:

- A solution to an open-ended question done as homework that demonstrates originality and unusual procedures
A mathematical autobiography
Teacher-completed checklists
Work in the students’ primary language
Student- or teacher-written notes from an interview
Papers that show the student’s correction of errors or misconceptions
A photo or sketch of student’s work with manipulatives or mathematical models of multi-dimensional figures
A letter from the student to the reader of the portfolio, explaining each item
A description by the teacher of a student activity that displayed understanding of a mathematical concept or relation
Draft, revised, and final versions of student work on a complex mathematical problem, including writing, diagrams, graphs, charts, or whatever is most appropriate
Excerpts from a student’s daily journal
Artwork done by students, such as string designs, coordinate pictures, scaled drawings, or maps
A problem made up by the student, with or without a solution
Work from another subject area that relates to mathematics, such as an analysis of data collected and presented in a graph for social studies
A report of a group project, with comments about the individual’s contribution, such as a survey of the use of mathematics in the world of work or a review of uses of mathematics in the media

Journals are a form of record keeping in which students respond in writing to specific probes or questions from the teacher. The probes focus student responses on knowledge or skill specific to a standard or benchmark. Journals of accomplishments can also be used informally to assess the development of writing skills. As with portfolios, whether a journal becomes an assessment tool depends upon how it is organized and evaluated.

Mrs. Becker asks her geometry students to keep journals in which they keep a record of the conjectures they make as a result of their geometric explorations. In her journal, Kimberly compares her conjectures with the theorems in the textbook and notates her questions. Mrs. Becker notices the question marks as she checks Kimberly’s journal and asks Kimberly to conference with her so that they can better assess the areas in which Kimberly has questions.
The Use of Assessment Rubrics

An assessment rubric is a set of rules used to rate a student’s proficiency on performance tasks (for example, essays, short-answer exercises, projects, and portfolios). Rubrics can be thought of as scoring guides that permit consistency in assessment activities. A rubric often consists of a fixed scale describing levels of performance and a list of characteristics describing performance for each of the points on the scale. Rubrics provide important information to teachers, parents, and others interested in what students know and can do. Most often, scoring rubrics are developed by a teacher or team of teachers, but it may be desirable in some instances to involve students in the creation of the rubrics. Different scoring rubrics are usually developed for each assessment activity, although if the activities are similar enough, a single rubric can be applied.

For an example of a carefully developed six-point scoring rubric for use in a writing performance assessment, see the Florida Writes rubrics at the end of this chapter and see publications describing the Florida Writes statewide assessment program. Less formal rubrics that might be used with a high school classroom assignment are shown in the following example:

Mr. Freer presents his high school students with this problem:

Mrs. Fariba Suarez has decided to use the profits from her swimwear company to open a teen night club. After studying much statistical data, Fariba surmises that this club should seat a minimum of 1152 people at tables. The tables will be 3 feet by 6 feet, and each one will seat 6 customers. Fire codes require that all tables be at least 3 feet from the wall and that there be at least a 3-foot space between tables. There must be a 6-foot-wide walkway through the center of the room parallel to the width of the building. Fariba must now decide on the dimensions of the building. Assuming the length of the building will be twice the width, find the dimensions of the smallest possible building Fariba can construct and describe the arrangement of tables in the building.

Mr. Freer wants his students to demonstrate their ability to use creative problem-solving processes and communicate their understanding of mathematical concepts. For the class assignment, students break into groups to develop a strategy for solving the problem. Each group will create a graphic representation of its problem-solving process and present its solution along with the problem-solving strategy to the class. After the presentation, each group will complete a final written report.
Mr. Freer creates a four-category checklist to be used to monitor whether each student performs all required dimensions of the assignment. He also creates four-point scoring rubrics to evaluate the proficiency of each student’s class presentation and written report.

The simple checklist might look like this:

<table>
<thead>
<tr>
<th>Student Name &amp; Date:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem-solving strategy and solution created on schedule?</td>
</tr>
<tr>
<td>Graphic representations completed?</td>
</tr>
<tr>
<td>Presentation delivered to class?</td>
</tr>
<tr>
<td>Paper completed on time?</td>
</tr>
</tbody>
</table>

**Teacher Rubrics**

Three simple five-point scoring rubrics are presented below and on the following page as examples of how teachers might evaluate three important elements of the experiment and the classroom presentation. These rubrics have specific descriptions only at the extremes and mid-point. A “4” and a “2” can be used to indicate performances that fall between these extremes.

Element 1, Approaches to Problem Solving and Reasoning

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows unique approaches to solving the problem; provides a complete solution to the problem; provides clear graphic models of the problem; uses known facts, properties, and relationships to explain the thinking used to solve the problem.</td>
<td>5</td>
</tr>
<tr>
<td>Demonstrates the use of problem-solving and reasoning strategies; provides a correct answer to the problem.</td>
<td>4</td>
</tr>
<tr>
<td>Solution is not understandable; solution is provided without explanation; solution does not demonstrate problem-solving and reasoning processes.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
Element 2, Material Content

<table>
<thead>
<tr>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrates a thorough understanding of mathematical concepts and facts related to the problem; uses additional information to extend the solution; effectively explains relationships among different topics within mathematics and/or other subject areas.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

| Displays a complete and accurate understanding of mathematical concepts and facts related to the problem. |

<table>
<thead>
<tr>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

| Demonstrates severe misconceptions about mathematical concepts and facts related to the problem. |

<table>
<thead>
<tr>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

Element 3, Presentation of Solution

<table>
<thead>
<tr>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responds with a coherent, concise, unambiguous explanation; communicates effectively to intended audience.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

| Some disorganization of content or some content omitted; audience generally interested in flow of ideas. |

<table>
<thead>
<tr>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

| Little relevant content; disorganized; difficult to understand; audience not interested. |

<table>
<thead>
<tr>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

Similar scoring rubrics would be necessary to evaluate the written report required of each student.
Student Rubrics

Students may also be asked to evaluate their own presentations. The rubrics created by the teacher can be rewritten as self-assessment rubrics for students so that students have the opportunity to evaluate their own performances on a scale similar to their teacher’s. The three student self-assessment rubrics presented below and on the following page have been modified from the above rubrics.

Element 1, Approaches to Problem Solving and Reasoning

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>I gave a thorough explanation of my approach to solving the problem; I provided a solution to the problem; I created clear graphic models of the problem; I used the facts, properties, and relationships I knew to explain how I solved the problem.</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>I gave a correct answer to the problem and explained my approach to reaching my solution.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>My solution wasn’t correct and I couldn’t really explain how I came to it.</td>
<td>1</td>
</tr>
</tbody>
</table>
### Element 2, Material Content

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>I understand mathematical concepts and facts related to the problem; I know how this knowledge could be used to solve other kinds of problems both in mathematics and in other subject areas.</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>I understand the mathematical concepts and facts related to the problem.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>I don’t really understand the mathematical concepts and facts related to the problem.</td>
<td>1</td>
</tr>
</tbody>
</table>

### Element 3, Presentation of Solution

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>My presentation was well organized and easy to follow; I gave a clear explanation of my solution and how I reached it; I maintained the interest of the audience.</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>My presentation was sometimes disorganized; I forgot to include some information; the audience was generally interested in my flow of ideas.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>My presentation was disorganized; I didn’t really explain my solution or how I reached it; the audience didn’t seem to be interested.</td>
<td>1</td>
</tr>
</tbody>
</table>
The Florida Writes Rubrics

Another kind of rubric is used by the Florida Writes writing assessment program to assess the quality of student writing. Teachers can use this rubric to assess writing in the language arts classroom and to prepare students for success on the state writing assessment. These rubrics are presented on the following pages.
Florida Writes Rubric: Grade 4

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Points</td>
<td>The writing is focused on the topic, has a logical organizational pattern (including a beginning, middle, conclusion, and transitional devices), and has ample supporting ideas or examples. The paper demonstrates a sense of completeness or wholeness. The writing demonstrates a mature command of language, including precision in word choice. Subject/verb agreement and verb and noun forms are generally correct. With few exceptions, the sentences are complete, except when fragments are used purposefully. Various kinds of sentence structures are used.</td>
</tr>
<tr>
<td>5 Points</td>
<td>The writing is focused on the topic with adequate development of supporting ideas. There is an organizational pattern, although a few lapses may occur. The paper demonstrates a sense of completeness or wholeness. Word choice is adequate but may lack precision. Most sentences are complete, although a few fragments may occur. There may be occasional errors in subject/verb agreement and in standard forms of verbs and nouns, but not enough to impede communication. The conventions of punctuation, capitalization, and spelling are generally followed. Various kinds of sentence structures are used.</td>
</tr>
<tr>
<td>4 Points</td>
<td>The writing is generally focused on the topic, although it may contain some extraneous or loosely related information. An organizational pattern is evident, although lapses may occur. The paper demonstrates a sense of completeness or wholeness. In some areas of the response, the supporting ideas may contain specifics and details, while in other areas, the supporting ideas may not be developed. Word choice is generally adequate. Knowledge of the conventions of punctuation and capitalization is demonstrated, and commonly used words are usually spelled correctly. There has been an attempt to use a variety of sentence structures, although most are simple constructions.</td>
</tr>
<tr>
<td>3 Points</td>
<td>The writing is generally focused on the topic, although it may contain some extraneous or loosely related information. Although an organizational pattern has been attempted and some transitional devices have been used, lapses may occur. The paper may lack a sense of completeness or wholeness. Some supporting ideas or examples may not be developed with specifics and details. Word choice is adequate but limited, predictable, and occasionally vague. Knowledge of the conventions of punctuation and capitalization is demonstrated, and commonly used words are usually spelled correctly. There has been an attempt to use a variety of sentence structures, although most are simple constructions.</td>
</tr>
</tbody>
</table>
Florida Writes Rubric: Grade 4 (continued)

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Points</td>
<td>The writing may be slightly related to the topic or may offer little relevant information and few supporting ideas or examples. The writing that is relevant to the topic exhibits little evidence of an organizational pattern or use of transitional devices. Development of supporting ideas may be inadequate or illogical. Word choice may be limited or immature. Frequent errors may occur in basic punctuation and capitalization, and commonly used words may be frequently misspelled. The sentence structure may be limited to simple constructions.</td>
</tr>
<tr>
<td>1 Point</td>
<td>The writing may only minimally address the topic because there is little, if any, development of supporting ideas, and unrelated information may be included. The writing that is relevant to the topic does not exhibit an organizational pattern; few, if any, transitional devices are used to signal movement in the test. Supporting ideas may be sparse, and they are usually provided through lists, clichés, and limited or immature word choice. Frequent errors in spelling, capitalization, punctuation, and sentence structure may impede communication. The sentence structure may be limited to simple constructions.</td>
</tr>
</tbody>
</table>
| Unscorable | The paper is UNSCORABLE because  
  - the response is not related to what the prompt requested the student to do.  
  - the response is simply a rewording of the prompt.  
  - the response is a copy of a published work.  
  - the student refused to write.  
  - the response is illegible.  
  - the response is incomprehensible (words arranged in such a way that no meaning is conveyed).  
  - the response contains an insufficient amount of writing to determine if the student was attempting to address the prompt.  
  - the writing folder is blank. |
Florida Writes Rubric: Grade 8

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Points</td>
<td>The writing is focused, purposeful, and reflects insight into the writing situation. The paper conveys a sense of completeness and wholeness with adherence to the main ideas, and its organizational pattern provides for a logical progression of ideas. The support is substantial, specific, relevant, concrete, and/or illustrative. The paper demonstrates a commitment to and an involvement with the subject, clarity in presentation of ideas, and may use creative writing strategies appropriate to the purpose of the paper. The writing demonstrates a mature command of language (word choice) with freshness of expression. Sentence structure is varied, and sentences are complete except when fragments are used purposefully. Few, if any, convention errors occur in mechanics, usage, and punctuation.</td>
</tr>
<tr>
<td>5 Points</td>
<td>The writing focuses on the topic, and its organizational pattern provides for a progression of ideas, although some lapses may occur. The paper conveys a sense of completeness or wholeness. The development of the support is ample. The writing demonstrates a mature command of language, including precision in word choice. There is variation in sentence structure, and, with rare exceptions, sentences are complete except when fragments are used purposefully. The paper generally follows the conventions of mechanics, usage, and spelling.</td>
</tr>
<tr>
<td>4 Points</td>
<td>The writing is generally focused on the topic but may include extraneous or loosely related material. An organizational pattern is apparent, although some lapses may occur. The paper exhibits some sense of completeness or wholeness. The support, including word choice, is adequate, although development may be uneven. There is little variation in sentence structure, and most sentences are complete. The paper generally follows the conventions of mechanics, usage, and spelling.</td>
</tr>
<tr>
<td>3 Points</td>
<td>The writing is generally focused on the topic but may include extraneous or loosely related material. An organizational pattern has been attempted, but the paper may lack a sense of completeness or wholeness. Some support is included, but development is erratic. Word choice is adequate but may be limited, predictable, or occasionally vague. There is little, if any, variation in sentence structure. Knowledge of the conventions of mechanics and usage is usually demonstrated, and commonly used words are usually spelled correctly.</td>
</tr>
</tbody>
</table>
Florida Writes Rubric: Grade 8 (continued)

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Points</td>
<td>The writing is related to the topic but includes extraneous or loosely related material. Little evidence of an organizational pattern may be demonstrated, and the paper may lack a sense of completeness or wholeness. Development of support is inadequate or illogical. Word choice is limited, inappropriate, or vague. There is little, if any, variation in sentence structure, and gross errors in sentence structure may occur. Errors in basic conventions of mechanics and usage may occur, and commonly used words may be misspelled.</td>
</tr>
<tr>
<td>1 Point</td>
<td>The writing may only minimally address the topic. The paper is a fragmentary or incoherent listing of related ideas or sentences or both. Little, if any, development of support or an organizational pattern or both is apparent. Limited or inappropriate word choice may obscure meaning. Gross errors in sentence structure and usage may impede communication. Frequent and blatant errors may occur in the basic conventions of mechanics and usage, and commonly used words may be misspelled.</td>
</tr>
</tbody>
</table>
| Unscorable | The paper is UNSCORABLE because  
- the response is not related to what the prompt requested the student to do.  
- the response is simply a rewording of the prompt.  
- the response is a copy of a published work.  
- the student refused to write.  
- the response is illegible.  
- the response is incomprehensible (words are arranged in such a way that no meaning is conveyed).  
- the response contains an insufficient amount of writing to determine if the student was attempting to address the prompt.  
- the writing folder is blank. |
Florida Writes Rubric: Grade 10

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Points</td>
<td>The writing is focused and purposeful, and it reflects insight into the writing situation. The organizational pattern provides for a logical progression of ideas. Effective use of transitional devices contributes to a sense of completeness. The support is substantial, specific, relevant, and concrete. The writer shows commitment to and involvement with the subject and may use creative writing strategies. The writing demonstrates a mature command of language with freshness of expression. Sentence structure is varied, and few, if any, convention errors occur in mechanics, usage, punctuation, and spelling.</td>
</tr>
<tr>
<td>5 Points</td>
<td>The writing is focused on the topic, and its organizational pattern provides for a logical progression of ideas. Effective use of transitional devices contributes to a sense of completeness. The support is developed through ample use of specific details and examples. The writing demonstrates a mature command of language, and there is variation in sentence structure. The response generally follows the conventions of mechanics, usage, punctuation, and spelling.</td>
</tr>
<tr>
<td>4 Points</td>
<td>The writing is focused on the topic and includes few, if any, loosely related ideas. An organizational pattern is apparent, and it is strengthened by the use of transitional devices. The support is consistently developed, but it may lack specificity. Word choice is adequate, and variation in sentence structure is demonstrated. The response generally follows the conventions of mechanics, usage, punctuation, and spelling.</td>
</tr>
<tr>
<td>3 Points</td>
<td>The writing is focused but may contain ideas that are loosely connected to the topic. An organizational pattern is demonstrated, but the response may lack a logical progression of ideas. Development of support may be uneven. Word choice is adequate, and some variation in sentence structure is demonstrated. The response generally follows the conventions of mechanics, usage, punctuation, and spelling.</td>
</tr>
<tr>
<td>2 Points</td>
<td>The writing addresses the topic but may lose focus by including extraneous or loosely related ideas. The organizational pattern usually includes a beginning, middle, and ending, but these elements may be brief. The development of the support may be erratic and nonspecific, and ideas may be repeated. Word choice may be limited, predictable, or vague. Errors may occur in the basic conventions of sentence structure, mechanics, usage, and punctuation, but commonly used words are usually spelled correctly.</td>
</tr>
</tbody>
</table>
Florida Writes Rubric: Grade 10 (continued)

| 1 Point | The writing addresses the topic but may lose focus by including extraneous or loosely related ideas. The response may have an organizational pattern, but it may lack a sense of completeness or closure. There is little, if any, development of the support, and the support may consist of generalizations or fragmentary lists. Limited or inappropriate word choice may obscure meaning. Frequent and blatant errors may occur in the basic conventions of sentence structure, mechanics, usage, and punctuation, and commonly used words may be misspelled. |
| Unscorable | The paper is UNSCORABLE because  
* the response is not related to what the prompt requested the student to do.  
* the response is simply a rewording of the prompt.  
* the response is a copy of a published work.  
* the student refused to write.  
* the response is illegible.  
* the response is incomprehensible (words are arranged in such a way that no meaning is conveyed).  
* the response contains an insufficient amount of writing to determine if the student was attempting to address the prompt.  
* the writing folder is blank. |
**Key Chapter Points**

- Assessment processes seek to measure students’ acquisition and application of skills and all aspects of knowledge and its connections.

- Assessment activities in the classroom should be integral, ongoing parts of the instruction and learning process.

- Teachers should use a variety of assessment methods and modifications to address different learning styles and student needs.

- Teachers have a wide variety of options for collecting information on the degree to which students have acquired and can apply knowledge and skills specific to language arts.

- Assessment activities will produce useful information to the degree that they are carefully planned, well organized, and consistently applied.

- Accurate assessment of student achievement provides a sound basis for classroom instructional decisions.
Chapter 7: The Learning Environment

Chapter Highlights

- Design of Facilities
- Safety
- Scheduling
- Learning Resources
- Selection of Materials
- Using Technology
- Snapshot of an Effective Mathematics Classroom

Goal 4: School boards provide a learning environment conducive to teaching and learning.

Florida’s System of School Improvement and Accountability

Twenty-first-century classrooms envisioned by Florida’s education reform initiative allow students to experience learning in its real-world context. These active learning environments extend beyond the four walls of the classroom into the home, the local community, and even the larger global community. Teachers are encouraged to incorporate more community projects and more interaction with their local communities. For example, teachers may provide opportunities for students to participate in job-shadowing programs with community leaders and members of the business community. Local citizens may be invited into classrooms to share knowledge, skills, or ideas, or to participate in classroom projects. Students may also have direct access to the global community via computers, satellite transmissions, teleconferencing, and other technology, enabling them to work with other students and experts across the state, in other states, or in other countries.
Design of Facilities

There are many factors to consider in designing a physical environment that facilitates the most effective learning. The ideal mathematics classroom is inviting. It has enough space for the free and flexible movement needed for a wide variety of learning approaches, such as cooperative learning, project work, and learning centers. Classroom furnishings may consist simply of tables and chairs, or desks and work areas that can be arranged and rearranged. The acoustics need to facilitate both classroom interaction and quiet time for reflection. Classrooms should have adequate storage and security for equipment and supplies; special consideration should be given to the proper storage of computers and other special equipment. In addition, classrooms should have appropriate technology support facilities, such as network access ports and electrical power outlets with ground-fault circuit interruption protection. Teachers also need a carefully designed space for research, planning, collaboration with other teachers, and reflection. The elements considered in the physical design of classrooms can apply in designing the teacher’s space as well.

Educators should become familiar with the legal requirements concerning students with disabilities (I.D.E.A. and Rehabilitation Act, Section 504), which state that classrooms must accommodate disabled students. The Americans with Disabilities Act describes people as disabled if they have a physical or mental impairment that substantially limits one or more activities. There are many possible adaptations to the classroom, hallway, cafeteria, vocational workshop, or other areas of the school that can meet the needs of students with disabilities. These might include ramps, elevators, and raised work spaces for students who use wheelchairs; sound-absorbing materials to reduce reverberation for hearing-impaired students; and sufficient lighting for students with visual impairment.

Local school districts have many factors to consider when evaluating what is needed for the design or redesign of facilities. These factors might include local needs and goals, budgets, instructional methods, adaptations to meet the needs of individual students, potential changes in student enrollment, and flexibility to allow for changes to meet new conditions in the future.
Safety

Goal 5: Communities provide an environment that is drug-free and protects students’ health, safety, and civil rights.

Florida’s System of School Improvement and Accountability

Schools should incorporate safety and health practices into the school environment. A safe, secure, learning environment for all students is an essential responsibility of the whole school community. Manuals specifying safety policies and regulations and incorporating state and federal policies are available for local schools. One aspect of school safety involves the physical environment, which should provide safe, clean facilities that meet all legal requirements. The environment should be free of odors, allergens, and harmful chemicals such as asbestos. To provide safety in the physical environment for students with disabilities, adaptations may be necessary, such as flashing fire alarms and special procedures for evacuation. A second aspect of school safety involves the supervision of students. Teachers must be aware of and understand safety procedures inside the school building, on school grounds, on field trips, and at special school events. Class activities conducted away from the classroom need to be carefully planned and examined for possible hazards. A third aspect of safety involves providing an environment in which everyone is safe from verbal, physical, and psychological harm. Teachers should also be prepared to use strategies for crisis intervention and conflict resolution.

Scheduling

Adequate time is essential for quality instruction and learning in order for students to achieve high academic standards. Students need sufficient time for concentrated involvement in learning experiences or projects. They may need time for extended discussions, experimentation, comprehension, and reflection.

Florida’s education reform initiative envisions that a strong element of the school improvement process will be provided by the local school community. This will have a significant effect on teachers’ work schedules and on the time teachers spend in pre-planning, instruction, assessment, and evaluation of classroom activities. For example, professional educators will need time to research new instructional approaches and to further develop integrated, meaningful lesson plans. Teachers may
need additional time for selecting teaching materials, designing student assessment strategies, and structuring specific learning experiences. Time must also be available for conferencing with other teachers, counselors, psychologists, and administrators, and for communicating with parents.

Another aspect of scheduling involves the range of teacher responsibilities and class size, both of which can have a significant impact on the classroom environment. No single formula is adequate to determine the appropriate work load for teachers or the appropriate class size for all schools and districts. Generally, an acceptable range is established at the district level, taking into consideration the characteristics of the unique student population, the composition of individual classes, funding levels, current and planned education reforms, extra duties and activities teachers undertake, and the organization and administration of the school.

To increase the effectiveness of the way time is used for teaching and learning, local school districts and schools are investigating ways to amend their present time structures. For example, educators are using block scheduling, year-round calendars, combined courses, and other strategies.

Learning Resources

Classrooms today are alive with activity and use a broad range of resources: from simple construction paper and crayons, baby food jars, buttons and other manipulatives, newspapers, films, and textbooks to electronic encyclopedias, graphing calculators, equipment and software for teleconferencing and satellite transmissions, and sophisticated laboratory devices. There may be colorful displays on the walls, maps to pull down, globes to touch, and a variety of primary and secondary source materials. The availability of appropriate manipulative materials and technological tools increases the mathematics teacher’s ability to move towards more active learning in the mathematics classroom. Computer stations with multimedia capabilities, software, and up-to-date instructional materials are used to encourage active and authentic learning and assist in research and in the production of learning projects.

Instructional materials, assistive technology, and equipment are available for students with a variety of special needs. For example, for students with visual impairment, Braille and large-print books can be obtained through the Florida
Instructional Materials Center. Adaptive computers, low-vision optical aids, and print-enlarging equipment are also available for vision-impaired students. Close-captioned videos for students with hearing impairments are developed at the Florida School for the Deaf and the Blind. As with instructional modifications, these specialized materials can often benefit students with learning difficulties who do not qualify for exceptional-student education programs.

**Selection of Materials**

The careful selection of instructional materials that support the development of conceptual understanding and encourage active learning is critical to a successful mathematics program. Teachers play a central role in the selection of instructional materials both for the overall school and for their classrooms. Whenever possible, teachers should collaborate to consider books, resources, and other major purchases for the school or district.

In developing their instructional plans, teachers consider a wide range of materials for use in their classrooms. In addition to textbooks, useful materials include manipulatives, supplementary trade books, reference materials, posters, supplies, audiovisual materials, computer software, and multimedia materials and supplies. Teachers should base their selection of classroom materials on the instructional plan and the specific needs of the students. They might examine the content and presentation of the materials from many different perspectives, including the vision and goals of the local school, the goals of their specific instructional plan, and the school budget. Educators should refer to state guidelines and district policies as possible resources for evaluating and selecting specific materials.

**Using Technology**

New technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them. (NCTM, 1989)

The use of technology is already changing the world of business and industry and is transforming our schools as well. Because technology is such a powerful tool for teachers and students, opportunities for training in its use and applications should be a part of
all education programs. Achieving high levels of skill in the use of technology will help students reach Florida’s high academic standards and contribute to their success in the workplace.

Technology can transform the classroom/laboratory into a multimedia learning center, giving teachers and students access to word processing, presentation tools, graphics, media integration, desktop publishing, and telecommunications resources. The application of technology in a mathematics classroom can benefit students in a multitude of ways. For example, it can

- give students more control and involvement in their own learning process;
- promote investigative skills;
- serve as an access to almost unlimited sources of information;
- provide students with skills to measure, monitor, and improve their own performance and develop competencies for the workplace;
- make learning more interesting for students;
- enable students to communicate with people from many parts of the world, bringing the sights, sounds, and thoughts of another language and culture into the classroom;
- provide opportunities to apply knowledge to simulated or real-life projects; and
- prepare students for a high-tech world of work.

Technological tools are a key element in a mathematics learning environment; they enable schools to provide a richer set of mathematical investigations and problem-solving experiences for all students. Educational technology used for mathematics activities include many tools, such as computers, LCD panels, video discs, CD-ROMs, calculators, calculator-based laboratories, and modems. Calculators and computers give students, engaged in solving mathematical problems, the opportunity to expand the scope of their exploration of mathematical ideas, models, structures, and simulations, and to increase the depth of their understanding of mathematical concepts. With the aid of technological tools, students can pose more challenging questions about the world around them and increase their ability to reason mathematically, use mathematical language, and make mathematical connections. It is important that students and teachers have as much access to these tools as possible for instructional and assessment activities at school and at home.
Distance learning uses communications technology to bring teaching and learning together through the transmission of information or expertise from one location to another. The use of this technology allows students to interact directly with teachers, experts, and students outside of their community.

Distance learning technologies are a valuable resource for mathematics education; they can enrich and enhance the learning experience for all students. Using the same technology that distributes most broadcast and cable TV signals, satellite-based distance learning services can reach hundreds or thousands of receiving sites located all over the United States. Some cable companies have developed services targeted specifically to educators and students. Through microwave systems and fiberoptic cables, distance learning programming can be more readily distributed to remote areas. Educators with computers and modems have access to an increasingly large selection of on-line data resources and dial-up bulletin boards. These services typically offer electronic mail, research databases, forums, and discussion groups for a variety of special interests.

Using telecommunications, students in Clearwater can exchange ideas with students in Ocala, Miami, and Pensacola, in other communities across Florida, in other states, and in other countries. Students can participate in a study or presentation in another location and students can talk to mathematicians working in the field. A class of fifth graders will better understand the geometry of a geodesic dome by asking questions as they watch an architect draw the architectural design or watch a builder construct a geodesic dome. These examples are not futuristic visions. They are typical experiences happening right now in schools across the country.

One technological tool that promises to have innovative applications in future classrooms is the use of live interactive video over an electronic on-line network. This technology can provide opportunities for students to take electronic “field trips” to the bottom of the ocean, to the rain forests, to the Arctic, or to outer space.
As technology evolves, it will be essential to evaluate which new tools will be most useful in the educational setting, given program goals, ever-expanding student needs, and existing equipment. Educators will need to keep up with the variety of technologies and their applications. New equipment and software programs become available at a rapid rate; the best choice for today may be quickly outmoded. Therefore, any recommendations for specific hardware or software programs should be flexible, forward thinking, and based on extensive research so that money will be well spent. In addition, teachers must make a commitment to become personally adept in using educational technology. They will need to add to and refine their skills on a regular basis by keeping up with new technological developments and exploring additional capabilities of current technology. Appropriate training and support opportunities should be established by administrators for that purpose.

The age of technology affords educators a wealth of choices. As the use of technology expands into education, educators will have more opportunities to discover new ways to explore ideas and meet the diverse individual needs of students. The availability and appropriate use of technology is indispensable in developing programs that will prepare the students of today to face continuing advancements in the workplace and to meet the technological changes that will occur in the 21st century.

**Snapshot of an Effective Mathematics Classroom**

Mrs. Watson takes her high school mathematics students on a field trip to a physics laboratory where they will be able to see the effect of gravity when objects are dropped in a vacuum. Using mathematical terms, a physicist explains the effects of gravity in a vacuum in which there is no drag from air. The physicist also demonstrates the various technologies used in the laboratory and discusses how technology and mathematical concepts are used in laboratory projects. In the next class, the students use the velocity and acceleration data they collected from the experiments at the physics laboratory to generate graphs and create equations that demonstrate the concept of slope as the rate of change for the fall of various objects. Mrs. Watson’s students also discuss what they learned at the physics laboratory and how this knowledge might be applied to other situations.

The following week, a visitor to Mrs. Watson’s mathematics class is surprised to see several students standing on chairs throughout the room while others crouch near the floor, dropping different objects to the ground from a set of standard heights. Other students are using motion detectors to measure the velocity and the acceleration of the objects as they fall. The motion
detectors are connected to graphing calculators; the velocity and acceleration data are collected and stored directly in the calculators’ memories. Upon completion of their tests, the students retrieve the data stored in the graphing calculator and generate a graph for each dropped object that shows the object’s position relative to time. After linking the graphing calculator to a computer, the students print copies of their graphs and begin to look them over. They find that their equations differ from the ones obtained at the physics lab using a vacuum.

As Mrs. Watson leads a class discussion about why this would happen, many students say they were surprised that a phone book and a paper clip seemed to fall at the same rate. Still others say they were surprised when the graph showed that, in fact, the objects had fallen at slightly different rates. “Maybe there is something else going on here. Let’s try the same test with a sheet of paper and a paper clip,” says Mrs. Watson. “Which object will reach the ground first and why?” The students reassemble in their groups and conduct the test, once again using their motion detectors and graphing calculators to collect and store their information. One student notes that the sheet of paper fell at a much slower rate than the paper clip. “It almost seemed to ‘float’ on the air,” he said. “Obviously weight is not the only factor in how fast objects fall. What effect does air have on the pull of gravity?” The students use their equations to generate conclusions about the effects of gravity, weight, mass, and surface area on the equations modeling the fall of various objects.

Technology in the mathematics classroom provides the opportunity for students to apply mathematical concepts to a broader range of real-world activities. In this example, the use of motion detectors and graphing calculators in Mrs. Watson’s classroom allows students to conduct a series of tests, measure minute differences, and gather large amounts of data for analysis. The students were able to test inferences and simulate situations that would not otherwise be possible in the classroom.
KEY CHAPTER POINTS

• Community resources and the latest technology should be tapped to bring the world into the classroom, allowing students to encounter learning in real-world contexts.

• Effective facilities are carefully planned, taking into account changes in student enrollments, student abilities, budgets, instructional needs, and the goals of the mathematics program.

• A safe, secure, learning environment is a priority for all students.

• Time can be used creatively, as a flexible resource.

• Classrooms should be rich with learning resources that afford opportunities for observation, manipulation of objects, exploration, experimentation, and discussion.

• The careful selection of instructional materials that effectively support the development of conceptual understanding and encourage active learning is critical to a successful mathematics program.

• As technology expands into education, mathematics educators can discover new ways to explore mathematics ideas and meet the diverse, individual needs of students.
Chapter 8: Professional Development

CHAPTER HIGHLIGHTS

• The Importance of Professional Development
• Rethinking Professional Development
• Preservice Education
• Effective Professional Development
• The Commitment to Lifelong Learning
• Attributes of the Professional Educator

The Importance of Professional Development

Professional development is a continuous improvement process lasting from the time an individual enters the education profession until retirement. It encompasses the processes that educators engage in to initially prepare themselves, continuously update themselves, and review and reflect on their own performance. If educators are to successfully prepare students for the future, they must be prepared for the future themselves. Schools and districts must be committed to offering the highest quality professional development opportunities for their teachers.

Rethinking Professional Development

Just as knowledge and skill requirements are changing for Florida students, so, too, are those for Florida educators. The globalization of commerce and industry, the explosive growth of technology, and the expansion of mathematics knowledge demand that teachers continually acquire new knowledge and skills. The challenge for every avenue of professional development is to provide learning opportunities in which preservice teachers, as well as more experienced teachers, can develop or acquire the necessary knowledge and skills to deal with change and pursue lifelong learning.
Preservice Education

Preservice education encompasses the training, preparation, and courses that future teachers undertake before certification. Research in schools across the nation shows that a crucial component of restructuring education is the teacher preparation program. Preservice education must develop the capacity to facilitate student learning and to be responsive to student and community needs, interests, and concerns (Darling-Hammond, 1993). To that end, teacher education programs at the college or university level are encouraged to incorporate the following:

- courses that develop a broad base of competencies, content area knowledge, and experiences for graduates to bring to the teaching profession;
- courses that include both theory and practice in teaching a diversity of students, including students with special needs;
- courses that present practical, proven, up-to-date approaches to curriculum, instruction, and assessment;
- training in the ability to understand and nurture the academic, emotional, and physical development of students;
- experiences that develop effective communication, team-building, and conferencing skills;
- extensive and ongoing student-teaching experiences that are supervised by qualified teachers and college or university personnel; and
- recognition that effective teachers must continue to grow professionally throughout their careers and must be proactive in seeking resources, assistance, and opportunities for growth.

By reexamining beliefs about teaching and learning, education faculties can design and implement improved teacher education programs. The goals of any such program are to produce creative, motivated, knowledgeable, confident, and technologically literate beginning teachers committed to lifelong growth.

Effective Professional Development

The term “professional development” is defined in this framework as those processes that improve and enhance the job-related knowledge and skills of practicing
classroom teachers. Professional development provides the continuous, on-the-job training and education needed to improve teaching, and ultimately, student learning. Florida’s school improvement initiative encourages local districts and schools to assume greater responsibility for professional development programs tailored to serve local school improvement efforts. Those educators charged with the design of these programs are urged to reflect upon the following characteristics of useful professional development:

An effective professional development program actively engages educators in the improvement process.

One facet of Florida’s school improvement and accountability initiative is to encourage local teams of educators to identify needs and clarify goals, solve problems, plan programs, monitor them reflectively, and make necessary adjustments. Professional development programs are an ideal way for districts to empower teachers to share in the decision-making processes within their schools and districts. Planners of professional development programs should encourage teachers to actively analyze their work, identify any needs and gaps in knowledge and skill, and provide suggestions about which resources might best close these gaps. Once educators have identified strategies to make school and classroom improvements, administrators and planners should use teacher expertise, wherever possible, in the preparation and delivery of professional development programs to support these strategies.

An effective professional development program continually updates the teacher’s knowledge base and awareness.

Systemic reform requires that teachers incorporate new teaching methods and content to help students achieve Florida’s new rigorous academic standards. Consequently, professional development programs must provide teachers with opportunities to acquire a broad base of new subject-area knowledge and instructional strategies so that Florida educators are better equipped to implement strategies to improve schools and raise achievement.

Educators will also need ongoing training in the use of educational technology. Equally important, professional development program planners are encouraged to work with teachers in identifying changes in student diversity, needs, and
problems. If teachers are to successfully engage students in the learning process, they must understand students’ cultural and linguistic backgrounds and life circumstances. In addition, professional development programs will need to address the issue of change: how to incorporate and embrace change in the classroom and how systemic reform impacts teaching methods and curriculum planning.

**An effective professional development program establishes a collaborative environment based on professional inquiry.**

Effective professional development encourages knowledge sharing and other opportunities for teachers to share ideas and experiences. Professional development strategies are most likely to be successful when teachers are encouraged to reflect on their own practices, identify problems and possible solutions, share ideas about instruction, engage in scholarly reading and research, and try out new strategies in their classrooms. Thus, staff networking, clinical education partnerships with universities in peer coaching, and mentoring are important tools to incorporate into long-range professional development planning. Peer coaching offers a nonthreatening environment in which teachers can implement new techniques and ideas and receive feedback from colleagues. Mentoring can be especially beneficial to new teachers; this mutually rewarding relationship with an experienced educator might include an exchange of teaching materials and information, observation and assistance with classroom skills, or field-testing of new teaching methods.

**An effective professional development program is continuously improved by follow-up.**

Professional development is an ongoing process; it does not simply consist of isolated presentations given by an expert or consultant. Effective inservice includes introductory training as well as a plan for ongoing monitoring, enhancement, and follow-up of learning. Research corroborates the need for follow-up that continues long enough for new behaviors learned during
introductory training to be incorporated into teachers’ ongoing practice (Sparks and Loucks-Horsley, 1989). Planners can build this kind of reinforcement into professional development programs in a number of ways, including providing opportunities to practice new methods in coaching situations, arranging for ongoing assistance and support, and systematically collecting feedback from teachers.

**An effective professional development program is actively and continuously supported by administrators.**

Numerous studies (McLaughlin & Marsh, 1978; Stallings and Mohlman, 1981; Loucks and Zacchie, 1983; Fielding and Schalock, 1985; Loucks-Horsley et al, 1987) reveal that active support by principals and district administrators is crucial to the success of any improvement effort. This supportive role begins with leadership that places a high priority on professional development, promotes communication, and fosters a spirit of collegiality. It extends to the thoughtful allocation of resources, including time. Up-to-date materials, classroom equipment, and time for educators to pursue opportunities for professional development and to practice and implement new teaching strategies are essential to ongoing staff improvement efforts. As Judy-Arin Krupp (1991) suggests, schools should

develop a norm for growth…that says staff development is not here to correct defects but to offer opportunities for everyone in the system to grow. Next, we need to recognize that everyone grows differently. We ask, “How can I help you grow as an educator so that we can provide the best possible education for students in this school?” (page 3)

**The Commitment to Lifelong Learning**

Effective mathematics educators do not rely solely on inservice programs provided by their schools or districts. They take personal responsibility for planning and pursuing other development activities.

As self-directed learners, quality mathematics educators strive to gain new insights, improve their skills, and broaden their perspectives. They work at the school and district levels to create professional development experiences for themselves and their colleagues. They form alliances with supervisors, professional development
specialists, principals, and other educators across all grade levels. They seek out quality workshops and courses. They take advantage of courses offered through technologies, such as on-line learning, interactive videoconferences, satellite teleconferences, and other innovative approaches to their own education. They also engage in experiential learning opportunities, such as “job shadowing” in their discipline or other practical, real-world experience in the community.

A particularly useful tool for professional development in mathematics can be membership in professional organizations. Professional organizations specific to mathematics include the following:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091-1593

National Council of Supervisors of Mathematics
P.O. Box 10667
Golden, CO 80401

Florida Council of Teachers of Mathematics
Nicky Walker, 1996 President
Santa Rosa County Schools
603 Canal Street
Milton, FL 32570

Florida Association of Mathematics Supervisors
Sue Burns, 1996 President
Orange County Schools
4600 Ocean Beach Blvd., #505
Cocoa Beach, FL 32931

In addition to providing invaluable opportunities for idea sharing and networking with other teachers, many professional organizations also publish journals that feature the latest developments in the field, assess new strategies and methodologies, and highlight new career and training opportunities. One publication that may be particularly helpful to mathematics educators is Professional Standards for Teaching Mathematics (1991).
Attributes of the Professional Educator

The goal underlying any Florida professional development program is to prepare educators in the competencies needed to successfully implement Florida’s long-term education improvement initiative. Shortly after the creation of Florida’s school improvement and accountability initiative, the Education Standards Commission began a project to identify and validate those teacher competencies necessary to successfully implement this initiative. The Commission’s efforts focused on the preparation and proficiency of teachers in helping students achieve higher and more rigorous standards. The Commission identified twelve broad principles and key indicators that reflect the high performance standards required of Florida’s teachers. These “accomplished practices” are summarized below.

Diversity
The professional educator uses teaching and learning strategies that reflect each student’s culture, learning styles, special needs, and socioeconomic background.

Assessment
The professional educator uses assessment strategies (traditional and alternative) to assist the continuous development of the learner.

Planning
The professional educator plans, implements, and evaluates effective instruction in a variety of learning environments.

Human Development and Learning
The professional educator uses an understanding of learning and human development to provide a positive learning environment that supports the intellectual, personal, and social development of all students.

Learning Environments
The professional educator creates and maintains positive learning environments in which students are actively engaged in learning, social interaction, cooperative learning, and self-motivation.
Communication
The professional educator uses effective communication techniques with students and all other stakeholders.

Critical Thinking
The professional educator uses appropriate techniques and strategies that promote and enhance the critical, creative, and evaluative thinking capabilities of students.

Technology
The professional educator uses appropriate technology in teaching and learning processes.

Role of the Teacher
The professional educator works with various education professionals, parents, and other stakeholders in the continuous improvement of the educational experiences of students.

Continuous Improvement
The professional educator engages in continuous professional quality improvement for self and school.

Knowledge and Understanding
The professional educator demonstrates knowledge and understanding of the subject matter.

Ethics and Principles
The professional educator adheres to the Code of Ethics and Principles of Professional Conduct of the Education Profession in Florida.
Key Chapter Points

• Florida’s school improvement initiative calls on schools to assume greater responsibility for professional development programs.

• If educators are to successfully prepare students for the future, they must be prepared for the future themselves.

• Preservice education should provide education graduates with a broad base of knowledge and skills to facilitate student learning and to be responsive to student and community needs, interests, and concerns.

• Inservice education should continue these efforts in an environment that supports and sustains teachers as individuals and collaborators in the process of systemic reform.

• Professional development programs should be designed to encourage every member of the learning community—teachers, support staff, and administrators—in their pursuit of lifelong learning.

• The role of professional development is to assist educators in developing the accomplished practices necessary to successfully implement Florida’s education reform initiative.
Chapter 9: Mathematics Program Improvement

CHAPTER HIGHLIGHTS
• The Nature of School Improvement
• The Evaluation Process
• Planning Changes for Improvement
• The Implementation Process
• Taking the Next Step

Students in the fictitious community of Sunrise Bay study mathematics beginning in kindergarten and continuing through twelfth grade. However, the School Advisory Council at Sunrise Bay High School has learned that local businesses find many graduates do not have adequate mathematics problem-solving and analytical skills to succeed in the work force. Sunrise Bay’s High School Advisory Council recommends a Mathematics Improvement Team be established to review the curriculum and methods of instruction in light of the needs of the business community. The team includes representatives from the business community in addition to mathematics teachers, personnel from middle and elementary schools, the principal, teachers from a variety of disciplines, district program coordinators, university faculty, students, parents, and other community citizens.

The Nature of School Improvement

The primary goal of Florida’s school improvement and accountability initiative is to raise student achievement by returning the problem-solving processes in education to the people closest to the students. This vision of local control can become a reality when individual schools and districts embrace the responsibility of becoming well-informed about the school improvement process, which may be both schoolwide and specifically targeted toward a single program.

In Florida, School Advisory Councils are charged with leading the overall school improvement process by drafting annual plans for raising student achievement and
meeting the state education goals and standards in all subject areas. These councils are composed of educators, parents, and community members who are representative of the diverse population served by the school.

The components of the improvement process make up a continuous cycle that entails a thoughtful study of the school program. The improvement process includes the following components: evaluating the results of the existing program in terms of student achievement and identifying areas of concern or areas that need improvement; determining the desired reforms to be undertaken; and implementing and evaluating these reforms. These components of the school improvement process can be applied to subject-area programs as well, both at the district and school levels. This chapter highlights the steps of the improvement process and offers guidelines to local educators as they improve their mathematics programs.

The Evaluation Process

The Mathematics Improvement Team meets to discuss how local businesses might assist in helping Sunrise Bay students achieve and apply the mathematical skills and abilities necessary for success in the work force. For guidance in this process, committee members review the locally developed vision statement for Sunrise Bay mathematics programs, which highlights student understanding of the relevance of school activities beyond the classroom. The Mathematics Improvement Team agrees that Sunrise Bay mathematics programs should provide opportunities for students to assimilate new information and solve problems in unconventional ways. As the team members study mathematics curricula at different grade levels, teaching methods, and results of student assessments, they discover that Sunrise Bay students have not been given the opportunity to apply their skills to authentic business experiences and to become proficient in using information systems technology. The Improvement Team considers various ways in which businesses can form partnerships with schools so that students can gain exposure to the forms of mathematics used in business environments.

Regular program evaluation ensures that the school implements mathematics programs that raise the achievement of all students, identify and meet the needs of the local community, and focus on content that aligns with state standards. Program evaluation should include, not just inform, all people involved in and affected by the
program. To help facilitate this process, districts and schools are encouraged to create Mathematics Improvement Teams.

With the overriding goal of student achievement as a backdrop, one of the Mathematics Improvement Team’s first tasks should be to develop a list of questions or concerns about the mathematics program. These might be organized around the components of this framework; for example, the program’s vision, its reflection of Florida’s Goal 3 standards, its use of innovative instructional strategies, or its connection to other disciplines. The questions might address program purposes, goals, content, context, instructional strategies, assessment methods and results, resources, attitudes of staff and students toward mathematics, and connections to other disciplines. Questions or concerns might also focus on the unique needs of the school or the local community.

During the evaluation process, it is useful to gather data about a variety of dimensions of the mathematics program from as many sources as appropriate and as possible. Some evaluation methods may be informal, part of the day-to-day activity of teaching and learning; others may be more formal, yielding information gathered from a variety of sources, such as

- surveys, questionnaires, and interviews;
- school statistics (for example, enrollment in specific subjects and electives);
- student assessments;
- reports from external evaluators; and
- self-evaluations.

Once information has been collected, the Mathematics Improvement Team should interpret it within the context of the identified questions or concerns and make recommendations for changing the program in order to bring about improvement in identified areas. Team members can also use the data to identify additional questions and concerns.

The process of generating questions and concerns to guide the review of the mathematics program, analyzing existing data, reaching conclusions on which parts need changing, and thinking up and testing solutions encourages ownership and shared responsibility for ongoing program improvement. Districts and schools are
encouraged to promote and integrate, where appropriate, innovative ideas suggested by those people specifically affected by and involved in the improvements.

**Planning Changes for Improvement**

The Mathematics Improvement Team polls local businesses to determine how the schools and the community can work together to improve student performance in mathematics problem-solving and analytical skills. Many local businesses demonstrate an interest in becoming involved with mathematics classrooms through mentoring and job-shadowing programs. The managing editor of the local newspaper suggests beginning an internship program for high school students who are interested in how mathematics is applied in the business world. Several company leaders agree to assist teachers in setting up authentic activities related to their fields and offer to provide feedback to students during the assessment process. A teacher on the Mathematics Improvement Team suggests pairing interested businessmen and women with students for job-shadowing, tutoring, and mentoring activities. Another member suggests that this job shadowing include teachers. A computer software engineer suggests that the Improvement Team petition the school board for the funds to upgrade the technology laboratories in each of the schools. The Mathematics Improvement Team writes a comprehensive improvement plan incorporating these ideas, including information on available resources and schedules for implementation.

Once areas needing improvement have been identified, the Mathematics Improvement Team can investigate various solutions and then develop a plan to make and implement the changes that will bring about improvement. A clear vision of the desired results is vital. In general, the plan should include a time line and a division of responsibilities to help assure its completion. It should be flexible and include continuous internal monitoring to determine the effectiveness of the changes to be implemented. The plan should also identify the general elements that will be needed to implement improvements, when each might occur, who will be responsible for what, and what resources are needed. Finally, the plan should align with schoolwide improvement.

It is important to keep in mind that all the additional resources needed may not be readily available. It may take some reallocation, some creative acquisition, some modification of existing resources to "get the job done." An important part of the plan is monitoring the results of any changes. If changes are not producing intended improvements or if obstacles develop, other approaches can be tried.
Developers of school and district mathematics improvement plans may wish to consider the following questions as they create the plan for improvement:

- Are all the stakeholders involved in the process?
- Is there a consensus about what needs improvement as well as potential strategies to be undertaken?
- Have periodic checks been established to monitor implementation?
- Has a reasonable time line been set?
- Have measures of adequate progress been clearly defined?
- Are the necessary human and financial resources available to implement the plan?

An important component of the improvement process is gaining the support and endorsement of those administrators who have overall responsibility for providing the resources and services to promote and facilitate the necessary changes. Staff development, different forms of evaluation, and/or different ways of operating in school buildings and classrooms may be required. Thus, administrative support for any improvement plan is critically important.

Once finalized, the improvement plan may be shared with those essential support systems that operate outside of the professional education community. Parents and guardians, elected officials, business and industry leaders, and members of media organizations all have a stake in the school improvement process. By communicating planned program improvements to the public, schools and districts encourage the involvement of all education stakeholders in the processes and operations of education, which in turn fosters the development of a greater sense of community.

The Implementation Process

The Mathematics Improvement Team is impressed with the level of involvement by the Sunrise Bay school community. Several internship programs are in place with various companies, giving students the opportunity to participate in the daily operations of business. Mathematics classes make frequent field trips to local business facilities, where students can witness current business technologies in operation. Speakers from the community enhance the mathematics program, speaking on topics such as banking, accounting, engineering, construction, aviation, architecture, and the medical field. The school board has approved funding for a state-of-the-art technology lab in which students may explore advanced technology including engineering,
computer-assisted drafting, laser technology, and a space-flight simulator. Through the improvements planned by the Improvement Team and implemented by local educators and their business partners, the Sunrise Bay community witnesses its vision for mathematics programs in action.

Implementation is the stage when the vision for improvement becomes a reality. After the Mathematics Improvement Team has gained approval for its plan, it should begin to orchestrate and coordinate activities, strategies, and tactics at the school level. Implementation gives teachers and administrators opportunities to put into practice what they have learned during the improvement process and to work toward achieving the goals set forth in the mathematics program vision statement.

Program improvement necessitates change, which progresses through several stages. People may initially oppose a change until they get enough information to become comfortable. With time, the innovation may even be improved by the very people who were opposed to its implementation.

Taking the Next Step

The community of Sunrise Bay is proud of its schools. Students, teachers, parents, and businesses have worked together to improve mathematics learning. Performance has improved on a variety of authentic measures in classrooms, and standardized test scores have improved. Mathematics scores continue to be strong in the statewide assessment, High School Competency Test (HSCT), Grade 11. The availability of new technology and equipment in the Sunrise Bay schools has motivated the faculty and stimulated the use of innovative instructional strategies in mathematics classrooms. Through the partnerships developed between schools and businesses, students have demonstrated their understanding of the importance and the applicability of mathematics skills to the business environment. Local businesses have noticed an improvement in the mathematics and analytical skills of those students who work in the community after high school graduation.

As schools improve, so does the community. As the community changes, so does the district’s PreK-12 mathematics program. The process is cyclical, continuous, and mutually beneficial.

The cyclical process of evaluation, planning for improvement, implementing changes for improvement, and monitoring the results of those changes has a number
of benefits. It involves a broad representation of the local community. It allows for
continual improvements that incorporate advances in technology and gains in
knowledge associated with the instruction of mathematics. It provides the
opportunity to create programs that meet the unique needs of students, address
specific local issues and concerns, and align with state standards. Ultimately, an
ongoing improvement process helps ensure success for each and every Florida student
in meeting high academic standards.

**Key Chapter Points**

- In both business and industry and in public sector organizations, a collaborative
  process of sound and systematic program evaluation, planning for improvement,
  implementation of innovative strategies, and monitoring of results leads to
  success.

- The overall improvement process being implemented through each School
  Advisory Council can also be applied to the mathematics program at either the
district or school level.

- Change happens slowly and only in an environment that encourages innovative
  and proactive thinking.

- To be systemic and successful, school and district programs should be designed
  with care, include all those concerned about success in education, and provide
time for creativity, implementation, practice, reflection, revision, and renewal.
Glossary

**Absolute value:** The number of units a number is from 0 on a number line. The absolute value of both 4 and −4, written |4| and |−4|, is 4.

**Additive identity:** The number zero.

**Additive inverse:** The opposite of a number. 19 and −19 are additive inverses of each other.

**Algebraic order of operations:** The order in which operations are done when performing computations on expressions. Do all operations within parentheses or the computations above or below a division bar; find the value of numbers in exponent form; multiply and divide from left to right; add and subtract from left to right. $5 + 10 ÷ 2 − 3 \times 2$ is $5 + 5 − 6$, or $10 − 6$ which is 4.

**Analog time:** Time displayed on a timepiece having hour and minute hands.

**Algebraic expression:** A combination of variables, numbers, and at least one operation, such as $x + 7 = 13$.

**Associative property of addition:** For all real numbers a, b, and c, their sum is always the same, regardless of how they are grouped. In algebraic terms: $(a + b) + c = a + (b + c)$; in numeric terms: $(5 + 6) + 9 = 5 + (6 + 9)$.

**Associative property of multiplication:** For all real numbers a, b, and c, their product is always the same regardless of how they are grouped. In algebraic terms: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$; in numeric terms: $(5 \cdot 6) \cdot 9 = 5 \cdot (6 \cdot 9)$.

**Central tendency:** A measure used to describe data (e.g., mean, mode, median).
**Chance:** The possibility of a particular outcome in an uncertain situation.

**Commutative property of addition:** Two or more factors can be added in any order without changing the sum. In algebraic terms: \( a + b + c = c + a + b = b + a + c \); in numeric terms: \( 9 + 6 + 3 = 6 + 3 + 9 = 3 + 9 + 6 \).

**Commutative property of multiplication:** Two or more factors can be multiplied in any order without changing the product. In algebraic terms: \( a \cdot b \cdot c = b \cdot c \cdot a = c \cdot b \cdot a \); in numeric terms: \( 5 \cdot 6 \cdot 9 = 5 \cdot 9 \cdot 6 = 6 \cdot 5 \cdot 9 \).

**Complex numbers:** Numbers that can be written in the form \( a + bi \), where \( a \) and \( b \) are real numbers and \( i = \sqrt{-1} \).

**Composite number:** A whole number that has more than two whole-number factors. 10 is a composite number whose factors are 1, 10, 2, 5.

**Concrete representation:** A physical representation (e.g., graph, model).

**Congruent:** Two things are said to be congruent if they have the same size and shape.

**Customary system:** A system of weights and measures frequently used in the United States. The basic unit of weight is the pound, and the basic unit of capacity is the quart.

**Digital time:** Time displayed in digits on a timepiece.

**Digit:** A symbol used to name a number. There are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In the number 49, 4 and 9 are digits.

**Dilation:** The process of reducing and/or enlarging a figure.

**Distributive Property of Multiplication over Addition:** Multiplying a sum by a number gives the same results as multiplying each number in the sum by the number and then adding the products. In algebraic terms: \( ax + bx = (a + b)x \) and \( x(a + b) = ax + bx \); in numeric terms: \( 3 \cdot (4 + 5) = 3 \cdot 4 + 3 \cdot 5 \).
**Equation:** A mathematical sentence that uses an equals sign to show that two quantities are equal. In algebraic terms: \( a + b = c \); in numeric terms: \( 3 + 6 = 9 \).

**Equivalent forms:** Different forms of numbers (e.g., fraction, decimal, percent) that name the same number, for example, \( \frac{1}{2} = .5 = 50\% \).

**Estimate:** An answer that is close to the exact answer. An estimate in computation may be found by rounding, by using front-end digits, by clustering, or by using compatible numbers to compute.

**Exponents (exponential form):** The number that indicates how many times the base occurs as a factor. \( 2^3 \) is the exponential form of \( 2 \times 2 \times 2 \), with 2 being the base and 3 being the exponent.

**Expression:** A mathematical phrase that can include operations, numerals, and variables. In algebraic terms: \( 2l + 3x \); in numeric terms: \( 13.4 - 4.7 \).

**Factor:** A number that is multiplied by another number to get a product. A number that divides another number exactly. The factors of 12 are 1, 2, 3, 4, 6, 12.

**Fractal:** A geometric shape that is self-similar and has fractional dimensions. Natural phenomena such as the formation of snowflakes, clouds, mountain ranges, and landscapes involve patterns. The pictorial representations of these patterns are fractals and are usually generated by computers.

**Function:** A relationship in which the output value depends upon the input according to a specified rule. For example, with the function \( f(x) = 3x \), if the input is 7, the output is 21.

**Histogram:** A bar graph that shows the frequency of data within intervals.

**Identity property of addition:** Adding zero to a number does not change the number’s value. \( x + 0 = x \); \( 7 + 0 = 7 \); \( \frac{1}{2} + 0 = \frac{1}{2} \).

**Identity property of multiplication:** Multiplying a number by 1 does not change the number’s value. \( y \cdot 1 = y \); \( 2 \cdot 1 = 2 \).
Inequality: A mathematical sentence that shows quantities that are not equal, using $<, >, \leq, \geq, \text{or} \neq$.

Infinite: Has no end or goes on forever.

Integers: The numbers in the set \{… −4, −3, −2, −1, 0, 1, 2, 3, 4 …\}.

Inverse property of addition: The sum of a number and its additive inverse is 0. For example, $3 + (−3) = 0$.

Inverse property of multiplication: The product of a number and its multiplicative inverse is 1. In algebraic terms: For all fractions, $\frac{a}{b}$ where $a, b \neq 0$, $\frac{a}{b} \times \frac{b}{a} = 1$; in numeric terms: $3 \times \frac{1}{3} = 1$; the multiplicative inverse is also called reciprocal.

Inverse operations: Operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations. For example, $20 - 5 = 15$ and $15 + 5 = 20$; $20 \div 5 = 4$ and $4 \times 5 = 20$.

Irrational numbers: A real number that cannot be expressed as a repeating or terminating decimal (i.e., square roots of numbers that are not perfect squares, such as $\sqrt{13}$; $0.121121112\ldots$).

Limit: A number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number.

Linear equation: An equation that can be graphed as a line on the coordinate plane.

Matrices: A rectangular array of mathematical elements (as the coefficients of simultaneous linear equations) that can be combined to form sums and products with similar arrays having an appropriate number of rows and columns.

Mean: The sum of the numbers in a set of data divided by the number of pieces of data; the arithmetic average.

Median: The number in the middle (or the averages of the two middle numbers) when the data are arranged in order.
**Midpoint:** The point that divides a line segment into two congruent line segments.

**Mode:** The number or item that appears most frequently in a set of data.

**Multiples:** The numbers that result from multiplying a number by positive whole numbers, for example the multiples of 15 are 30, 45, 60, …

**Natural (counting) numbers:** The numbers in the set \{1, 2, 3, 4, …\}.

**Numeration:** The act or process of counting and numbering.

**Number theory:** The study of the properties of integers (e.g., primes, divisibility, factors, multiples).

**Ordered pair:** A pair of numbers that can be used to locate a point on the coordinate plane. An ordered pair that is graphed on a coordinate plane is written in the form: (x-coordinate, y-coordinate), for example, (8,2).

**Operations:** Any process, such as addition, subtraction, multiplication, division, or exponentiation, involving a change or transformation in a quantity.

**Patterns:** A recognizable list of numbers or items.

**Parallel lines:** Lines that are in the same plane but do not intersect.

**Permutation:** An arrangement, or listing, of objects or events in which order is important.

**Perpendicular lines:** Two lines or line segments that intersect to form right angles.

**Planar cross-section:** The area that is intersected when a two-dimensional plane intersects a three-dimensional object.

**Plot:** To locate a point by means of coordinates, or a curve by plotted points, and to represent an equation by means of a curve so constructed.
Power: A number expressed using an exponent. The power $5^3$ is read five to the third power, or five cubed.

Prime: A number that can only be divided evenly by two different numbers, itself and 1. The first five primes are 2, 3, 5, 7, 11.

Proof: The logical argument that establishes the truth of a statement. The process of showing by logical argument that what is to be proved follows from certain previously proved or accepted propositions.

Proportion: An equation that shows that two fractions (ratios) are equal. In algebraic terms: $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$, $d \neq 0$; in numeric terms: $\frac{3}{6} = \frac{1}{2}$, $3:6 = 1:2$.

Probability: The number used to describe the chance of an event happening. How likely it is that an event will occur.

Radical: An expression of the form $\sqrt[n]{a}$ for example, $\sqrt[3]{64}$, $\sqrt[3]{27}$

Range: The difference between the greatest number and the least number in a set of data. The set of output values for a function.

Rational number: A number that can be expressed as a ratio in the form $\frac{a}{b}$ where $a$ and $b$ are integers and $b \neq 0$, for example, $\frac{1}{2}$, $\frac{3}{5}$, $-7$, $4.2$, $\sqrt{49}$.

Ratio: A comparison of two numbers by division. The ratio comparing 3 to 7 can be stated as 3 out of 7, 3 to 7, 3:7, or $\frac{3}{7}$.

Real numbers: The set of numbers that includes all rational and irrational numbers.

Rectangular coordinate system: A system formed by the perpendicular intersection of two number lines at their zero points, called the origin. The horizontal number line is called the $x$-axis, and the vertical number line is called the $y$-axis. The axes separate the coordinate plane into four quadrants.

Recursive definition: A definition of sequence that includes the values of one or more initial terms and a formula that tells how to find each term of a sequence from previous terms.
**Reflexive property:** A number or expression is equal to itself, as in $a = a$, $cd = cd$.

**Reflection:** The figure formed by flipping a geometric figure about a line to obtain a mirror image.

**Right triangle trigonometry:** Finding the measures of missing sides or angles of a triangle given the measures of the other sides or angles.

**Rotation:** A transformation that results when a figure is turned about a fixed point a given number of degrees.

**Scale:** The ratio of the size of an object or the distance in a drawing to the actual size of the object or the actual distance.

**Scientific notation:** A short-hand way of writing very large or very small numbers. The number is expressed as a decimal number between 1 and 10 multiplied by a power of 10, for example $7.59 \times 10^5 = 759,000$.

**Sequences:** An ordered list of numbers with either a constant ratio (geometric) or a constant difference (arithmetic).

**Series:** An indicated sum of successive terms of an arithmetic or geometric sequence.

**Similar:** Objects or figures are similar if their corresponding angles are congruent and their corresponding sides are in proportion. They are the same shape, but not necessarily the same size.

**Surface area:** The sum of the areas of all the faces of a three-dimensional figure.

**Symmetry:** The correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point.

**Tessellation:** A repetitive pattern of polygons that covers an area with no holes and no overlaps, like floor tiles.
**Transformation**: An operation on a geometric figure by which each point gives rise to a unique image. Common geometric transformations include translations, rotations, and reflections.

**Translation (also called a slide)**: A transformation that results when a geometric figure is moved by sliding it without turning or flipping it. Each of the points of the figure move the same distance in the same direction.

**Variable**: A symbol, usually a letter, used to represent one or more numbers in an expression, equation, or inequality. For example, in $5a; 2x = 8; 3y + 4 \neq 10$, $a$, $x$, and $y$ are variables.

**Whole numbers**: The numbers in the set {0, 1, 2, 3, 4, ...}.


